

Cocker's ARITHMETICK;

B E I N G

A plain and familiar Method, suitable to the meanest Capacity, for the full Understanding of that incomparable Art, as it is now taught by the ablest School-Masters in CITY and COUNTRY,

C O M P O S E D

By EDWARD COCKER, late Practitioner in the Arts of Writing, Arithmetick, and Engraving: Being that so long since promised to the World.

Perused and Published,

By JOHN HAWKINS, Writing-Master near St. George's Church in *Southwark*, by the Author's correct Copy, and commended to the World by many eminent Mathematicians and Writing Masters in and near *London*.

The FORTY-FIFTH EDITION, carefully
Corrected and Amended

By GEORGE FISHER, *Account.*
Licensed Sept. 3. 1677. Roger L'Estrange.

L O N D O N :

Printed for EDWARD MIDWINTER, at the
Three Crowns and Locking-Glass in St. Paul's
Church-Yard.

(1677)

TO his much honoured Friends *Manwaring Davies* of the *Inner Temple*, Esq; and *Mr. Humphry Davies* of *St. Mary Newington-Butts*, in the County of *Surry*.

John Hawkins, as an Acknowledgment of unmerited Favours) humbly dedicateth this *Manual of Arithmetick*.

To the R E A D E R.

Courteous Reader,

I Having had the Happiness of an intimate Acquaintance with *Mr. Cocker* in his Life-time, often solicited him to remember his Promise to the World, of publishing his *Arithmetick*; (but for Reasons best known to himself) he refused it; and after his Death) the Copy falling accidentally into my Hand) I thought it not convenient to smother a Work of so considerable a Moment, not questioning but it might be as kindly accepted, as if it had been presented by his own Hand. The Method is familiar and easy, discovering as well the Theorick as the Practick of that necessary Art of *Vulgar Arithmetick*. And in this new Edition there are many remarkable Alterations for the Benefit of the Teacher or Learner, which I hope will be very acceptable to the World: I have also performed my Promise, in Publishing the *D-cimal Arithmetick*, which finds Encouragement to my Expectation, and the Booksellers too. I am
thine to serve thee.

John Hawkins.



Mr

Edw. Cockers
Edw. Cockers
Mr. Edward Cocker's

PROEME or PREFACE.

BY the secret Influence of Divine Providence, I have been instrumental to the Benefit of many, by Vertue of those useful Arts, Writing and Engraving: And do now with the same wonted Alacrity, cast this my Arithmetical Mite into the publick Treasury, beseeching the Almighty to grant the like Blessing to these as to my former Labours.

Seven Sciences supremely excellent,
Are the chief Stars in Wisd:m's Firmament:
Whereof Arithmetick is one, whose Worth
The Beams of Profit and Delight shine forth;
This crowns the rest, this makes Man's Mind compleat,
This treats of Numbers, and of this we treat.

I have been often desired by my intimate Friends, to publish something on this Subject, who, in a pleasing Freedom, have signified to me, that they expected it would be extraordinary. How far I have answered their Expectation, I know not; but this I know, That I have designed this Work not extraordinary abstruse or profound; but have, by all Means possible, within the Circumference of my Capacity, endeavoured to render it extraordinary useful to all those, whose Occasions shall induce them to make use of Numbers. If it be objected, That the Books already published, treating of Numbers, are innumerable; I answer, That's but a small Wonder, since the Art is infinite. But that there should be so many excellent Tracts of Practical Arithmetick extant, and so little practised, is to me a great Wonder; knowing that as Merchandize is the Life of the Weal publick, so Practical Arithmetick is the Soul of Merchandize. Therefore I do ingenuously profess, That in the Beginning of this Undertaking, the numerous Concerns of the honoured

Merchants

The Proeme or Preface.

Merchant first, possesseth my Consideration. And how far I have accommodated this Composure for his most worthy Service, let his own profitable Experience be Judge.

Secondly, For your Service, most excellent Professors, whose Understandings soar to the Sublimity of the Theory and Practice of this noble Science, was this Arithmetical Treatise composed; which you may please to employ as a Monitor to instruct your young Tyroes, and thereby take Occasion to reserve your precious Moments, which might be exhausted that Way, for your more important Affairs.

Thirdly, For you the ingenious Off-spring of happy Parents, who will willingly pay the full Price of Industry and Exercise for those Arts and choice Accomplishments, which may contribute to the Felicity of your future State: For you, I say, (ingenious Practitioners) was this Work composed, which may prove the Pleasure of your Youth, and the Glory of your Age.

Lastly, For you the pretended Numerists of this Vapouring Age, who are more disingenuously witty to propound unnecessary Questions, than ingenuously judicious to resolve such as are necessary; for you was this Book composed and published, if you will deny your selves so much as not to invert the Streams of your Ingenuity, but by studiously conferring with the Notes, Names, Orders, Progress, Species, Properties, Proprieties, Proportions, Powers, Affections, and Applications of Numbers delivered herein, become such Artists indeed, as you now only seem to be. This Arithmetick ingeniously observed and diligently practis'd, will turn to good Account to all that shall be concerned in Accompts; since all its Rules are grounded on Verity, and delivered with Sincerity; the Examples built up gradually from the smallest Consideration to the greatest; and all the Problems or Propositions, well weighed, pertinent and clear, and not one of them throughout the Tract, taken upon Trust, therefore now,

Zoilus and Momus, lie you down and die,
For these Inventions your whole Force defie.

Edward Cocker.

Courteous READER,

Being well acquainted with the deceased Author, and finding him knowing and studious in the Myſteries of Numbers and Algebra, of which he had ſome choice Manuſcripts, and a great Collection of printed Authors in ſeveral Languages, I doubt not but he hath writ his Arithmetick ſuitable to his own Preface, and worthy Acceptation, which I thought fit to certify, on a Requeſt to that Purpoſe, made to him that wiſheth thy Welfare, and the Progreſs of Arts,

John Collens.

Novemb. 27. 1677.

This Manual of Arithmetick is recommended to the World by Us whoſe Names are ſubſcribed, viz.

Mr. John Collens	} Matth.	Mr. William Maſon
Mr. James Atkinſon		Mr. Stephen Thomas
Mr. Peter Perkins	} Sen.	Mr. Peter Storey
Mr. Rich. Lawrence,		Mr. Benj. Tichbourn
Mr. Eleazar Wigan		Mr. Joſeph Symmonds
Mr. Rich. Noble of Guilford		Mr. Jerem. Miles
Mr. William Norgate		Mr. Joſiah Cuffley
		Mr. John Hawkins

And generally Approved by all ingenious Artists.

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C H A P. I.

Notation of Numbers.

A *Rithmetick* is an Art of Numbering or Knowledge, which teacheth to Number well. And there are divers Species and Kinds of *Arithmetick* and *Geometry*, the which we do intend to treat of in order, applying the Principles of the one to the Definition of the other. For as Greatness is the Subject of *Geometry*, so Number is the Subject of *Arithmetick*; and if so, then their first Principles and chief Fundamentals must have like Definitions; or at least some Congruency.

2. Number is that, by which the Quantity of any Thing is express'd or number'd; as the Unite is the Number by which the Quantity of the Thing is express'd or said to be one, and two by which it is named two, and $\frac{1}{2}$ half, by which it is named or called half, and the Root of 3, by which it is called the Root of 3, the like of any other.

3 Hence it is that Unit is Number; for the Part is of the same Matter that is his Whole, the Unit is part of the Multitude of Units, therefore the Unit is of the same Matter, that is the Multitude of Units; but the Matter of the Multitude of Units is Number; therefore the Matter of Unit is Number; for else, if from a Number given no Number be subtracted, the Number given remaineth; as suppose 3 the given Number, if as some suppose, 1 be no Number, then if you subtract 1 from 3, there must remain 3 still; which is very absurd.

4. Hence it will be convenient to examine from whence Number hath its Rise or Beginning. Most Authors maintain, that Unit is the Beginning of Number, and it self no Number; but looking upon the Principles and Definitions in the first Rudiments of *Geometry*, we shall find that the

B

Definition

Definition of a Point is in no way congruous with the Definition of an Unit in Arithmetick; and therefore One or Unit must be in the Bounds or Limits of Number, and consequently the Beginning of Number is not to be found in the Number 1; wherefore making Number and Magnitude congruent in Principles, and like in Definitions, we make and constitute a Cypher to be the Beginning of Number, or rather the Medium between increasing and decreasing Numbers, commonly called absolute or whole Numbers, and negative and fractional Numbers, between which nothing can be imagin'd more agreeable to the Definition of a Point in Geometry; for as a Point is an Adjunct of Line, and it self no Line, so is a (0) Cypher an Adjunct of Number, and it self no Number: And as a Point in Geometry cannot be divided or increased into Parts; so likewise (0) cannot be divided or increased into Parts; for as many Points, though in Number infinite, do make no Line, so many (0) Cyphers, though in Number infinite, do make no Number.

For the Line A B cannot be increased by the Addition of the Point C, neither the Number D be increased by the Addition of the (0) Cypher E; for if you add Nothing to 6, the Sum will be 6, (0) Cypher neither increasing nor diminishing the Number 6; but if it be granted that A B be extended or prolonged to the Point C, so that A C be made a continued Line, then A B is increased by the Addition of the Point C. In like manner, if we grant D (6) be prolonged to E (0), so that D E (60) be a continued Number, making 60, then 6 is augmented by the

Aid of (0) as constituting the Number (60) Sixty; and furthermore that 1 or Unit is material and a Number, and that (0) is the Beginning of Number, is proved by all Authors, although directly; for the Tables of Signs and Tangents prove one Degree to be a Number, because the Sine of 1 Degree is 174524, (the Radius being 10000000) and the Beginning of the Table is (0), and it answereth 00000, &c.

A — — B

C

D 6

E 0

—

Sum 6

d

A — B — C

D E | 60

6 0 |

De- or on- in- ni- we- of- and- hoie- reen- fini- unct- unct- in- ; fo- rts- take- in- —B- 6- 6- — 6- B—C- 60- ; and- er, and- all Au- d Tan- he Sinc- (CCCC) (wereth- Hence

5 Hence it is that Number is not Quantity discontinued, for that which is but one Quantity, is not Quantity disjunct: (60) Sixty, as it is a Number, is one Quantity, viz. one Number (60) Sixty; therefore as it is a Number, it is not Quantity disjunct, for Number is some such Thing is Magnitude, as Humidity in Water: for as Humidity extends it self through all and every Part of Water, so Number related to Magnitude, do extend it self through all and every Part of Magnitude. Also, as continued Water doth answer continued Humidity, so to a continued Magnitude doth answer a continued Number. As the continued Humidity of an intire Water, suffereth the same Division and Distinction that his Water doth; so the continued Number suffereth the same Division and Distinction that his Magnitude doth. And thus much concerning the Definition and Principles of Number and Magnitude. We come now to treat of,

6. The Characters or Notes by which Numbers are signi'y'd, or by which a Number is ordinarily express'd; and they are these, viz. (0) Cypher or Nothing, 1 One, 2 Two, 3 Three, 4 Four, 5 Five, 6 Six, 7 Seven, 8 Eight, 9 Nine. The Cypher, which tho' of itself it expresseth not any certain or known Quantity, yet is the Beginning or Root of Number, and the other 9 Figures are called significant Figures or Digits.

7. In Number of any Sort, two Things are to be considered, viz. *Notation* and *Numeration*.

8. *Notation* teacheth how to describe any Number by certain Notes and Characters, and to declare the Value thereof being so described, that is by *Degrees* and *Periods*.

9. A Degree consists of three Figures, viz. of three Places, comprehending Units, Tens, and Hundreds, so 265 is a Degree, and the first Figure (5) on the right Hand, stands simply for his own Value, being Units, or so many Ones, viz. Five; the second in order from the Right, signifies as many times Ten as there are Units contained in it, viz. Sixty; the Third in the same Order signifies so many Hundreds as it contains Units, so will the Expression of the Number be Three hundred sixty five, &c.

10. A Period is when a Number consists of more than 3 Figures or Places, and whose proper Order is to prick

every third Place, beginning at the right Hand, and so on to the Left; so the Number 63452 being given, it will be distinguished thus, 63,452, and expressed thus, Sixty three thousand, four hundred fifty two; likewise 4,578,236,782, being distinguished as you see, will be expressed thus; Four thousand, five hundred seventy eight Millions, two hundred thirty six thousand, seven hundred eighty two.

11. Number is either Absolute or Negative.

12. Absolute or Intire, Whole, Increasing Number, is that by which annexing another Figure or Cypher, it becomes ten times as much as it stood for before; and if two Figures or Cyphers be annexed, it makes an hundred times as much as it stood for before, &c. As if you annex to the Figure 6 a Cypher, then it will be (60) Sixty; so if two Cyphers are annexed, then it will be (600) Six hundred, and if you do annex to it (4) Four, then it will be (64) Sixty four; and if you annex (78) Seventy eight, it will be then (678) Six hundred seventy eight, &c.

13. A Negative, or Broken, Fractional, Decreasing Number, is that which by prefixing a Point or Prick toward the left Hand, its Value has decreased from so many Units, to so many tenth Parts of any Thing; and if a Point and (0) Cypher, or Digit, be prefixed, it will be then so many hundred Parts; and if a Point and two Cyphers or Digits be prefixed, its Value is decreased to be so many thousandth Parts; as if you would prefix before the Figure 3 a Point (.) or Prick thus (.3), it is then decreased from 3 Units or 3 Integers, to 3 tenth Parts of an Unit or an Integer: And if you prefix a Point and Cypher thus (.03,) it is decreased from 3 Integers to 3 hundred Parts of an Integer; and by this Means 5 *l.* absolute, by prefixing of a Point, will be decreased to .5 *l.* negative, which is 5 tenth Parts of a Pound, equal in Value to 10 Shillings; and so by prefixing of more Cyphers or Digits, its Value is decreased in a decuple Proportion *ad infinitum*. As in the following Scheme, or rather Order of Numbers, we have placed (0) Cypher in its due Place and Order, as it is in the Beginning and Medium of Number; for going from (0) towards the left Hand, you deal with intire, absolute, whole, increasing Numbers.

Increasing Numbers.

Decreasing Numbers.

29	876	543	256	21012	345	678	976	3
mm	mmm	mmm	mmm	CXUXC	mmm	mmm	mmm	m
mm	mmm	mmm	CX		XC	mmm	mmm	m
mm	CX	CX				XC	mmm	m
mm							XC	
X								

But going from (o) the Place of Units towards the right Hand, you meet with broken negative, fractional and decreasing Numbers. And hence it follows, that *Multiplication* increaseth the Product in absolute Numbers, but decreaseth the Product in negative Numbers. Also *Division* decreaseth the Quotient in whole Numbers, and increaseth it in negative fractional Numbers.

14. An absolute, intire, whole, increasing Number, hath always a Point annexed towards the right Hand; and therefore,

15. A negative, broken, decimal, decreasing Number, hath always a Point prefixed towards the left Hand. When we express Integers or whole Numbers, as 5 Pounds, 5 Feet, 26 Men, we usually annex a Point or Prick after

l. feet. men. inch.

the Number thus,

5. 5. 26. 347.

But when we express Decimals, or Numbers that are denied to be intire, or decreasing Numbers, we do commonly prefix a Point or Prick before the said Decimal or decreasing Number thus (.3), that is three Tenths, or 3 Primes; (.03) that is 3 Hundredths, or 3 Seconds.

16. A whole or absolute Number is a Unit, or a composed Multitude of Units, and it is either a Prime or else a compound Number.

17. Prime Numbers amongst themselves, are those which have no Multitude of Units for a common Mesurer, as 8 and 7, or 10 and 13, because not any Multitude of Units can equally measure or divide them without a Remainder.

18. Compound Numbers amongst themselves, are those which have a Multitude of Units for a common Mesurer, as 9 and 12, because 3 measures them exactly, and abbreviates them to three and four.

19. A broken Number commonly called a Fraction, is a Part or Parts of a whole Number, viz. A Part of an Integer, as $\frac{1}{3}$ one Third, is one third Part of an Unit.

20. A broken Number or Fraction, consists of 2 Parts, viz. the Numerator and the Denominator.

21. The Numerator and Denominator of a Fraction, are set one over the other, with a Line between them; and the Numerator is set above the Line, and expresseth the Parts therein contained.

22. The Denominator of a Fraction, is the inferiour Number placed below the Line, and expresseth the Number of Parts into which the Unit or Integer is divided; as let $\frac{3}{4}$ be the Fraction given, so shall 3 be the Numerator, and doth expresse or number the Multitude of Parts contain'd in this Fraction, for $\frac{3}{4}$ is a Fraction compounded of Fourths or Quarters, and the Figure 3 in numbering shews us, that in that Fraction there are 3 of the 4th Parts or Quarters; also in the same Fraction $\frac{3}{4}$ is the Denominator, and doth expresse the Quality of the Fraction, viz. that the whole or Integer is divided into 4 equal Parts.

23. A broken Number is either Proper or Improper; viz. proper when the Numerator is less than the Denominator, for $\frac{3}{4}$ is a perfect proper Fraction; but an improper Fraction hath its Numerator greater, or at least equal to the Denominator, thus $1\frac{3}{4}$ is an improper Fraction, the Reason is in given in the Definition.

24. A proper broken Number, is either Simple or Compound, viz. Simple when it hath one Denomination, and Compound when it consisteth of divers Denominations; it $\frac{3}{4}$ $\frac{6}{12}$ $\frac{2}{5}$ were given, we say, they are each of them Single or Simple Fractions, because they consist but of one Numerator, and one Denominator; but if $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{2}{5}$ of a Pound Sterling were given, we say that it is a compound broken Number or Fraction, because the Expression and Representation consisteth of more Denominations than one; and such by some are called Fraction of Fractions; they have always this Particle (of) between them.

25. When a single broken Number or Fraction hath for his Denominator a Number consisting of a Unit in the first Place towards the left Hand, and nothing but Cyphers from the Unit towards the right Hand, it is then the more apply

aply, and rightly called a decimal Fraction ; under this Head are all our decreasing Numbers placed, and in our 13th Definition, called Negatives ; and by the Order there prescribed, we order them to be Decimals, by signing a Prick or Point before them, or the Numerator, rejecting the Denominator : Therefore according to our last Rule, $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, are said to be Decimals ; and a Decimal Fraction may be expressed without its Denominator (as before) by prefixing a Point or Prick before the Numerator of the said Fraction, and then shall the former Fractions $25 \frac{5}{10}$ and $18 \frac{5}{100}$ stand thus, .5, and .025.

But oftentimes, as in the second and fourth Fractions $\frac{5}{10}$ and $\frac{5}{100}$, a Prick or Point will not do without the Help of a Cypher or Cyphers prefixed before the significant Figures of the Numerator, and therefore when the Numerator of a decimal Fraction consisteth not of so many Places as the Denominator hath Cyphers, fill up the void Places of the Numerator with prefixing Cyphers before the significant Figures of the Numerator, and then sign for a Decimal, so shall $\frac{5}{10}$ be .05, and $\frac{5}{100}$ will be .025, and $\frac{72}{1000}$ will be .0072. Now by this we may easily discover the Denominator having the Numerator, for always the Denominator of any decimal Fraction consists of so many Cyphers, as the Numerator hath Places, with an Unit prefixed before the said Cypher, viz. under the Point or Prick.

26. A decimal Number or Fraction, is expressed by Primes, Seconds, Thirds, Fourths, &c. and is a Number decreasing. Here instead of natural and common Fractions, as $\frac{3}{4}$ of a Thing, we order the Thing or Integer into Primes, Seconds, Thirds, Fourths, Fifths, &c. that our Expression may be consonant to our former Order.

27. In decimal Arithmetick, we always imagine that all intire Units, Integers, and Things, are divided first into ten equal Parts, and these Parts so divided we call Primes ; and Secondly, we divide also each of the former Primes into other ten equal Parts, and every of these Divisions we call Seconds ; and Thirdly, we divide each of the said Seconds into ten other equal Parts, and those so divided, we call Thirds ; and so by decimating the former, and subdecimating these latter, we run on *ad infinitum*.

28. Let a Pound Sterling, Troy-weight, Averdupois-weight, Liquid measure, Dry-measure, Long-measure, Time, Dozen, or any other Thing or Integer be given to be decimally divided: In this Notion premised, we ought to let the first Division be Primes, the next Division Seconds, the next Thirds, &c. So one Pound Sterling being 20 Shillings, when divided into ten equal Parts, the Value of each Part will be 2 Shillings, therefore one Prime of a Pound Sterling will stand thus (.1), which is in Value 2 Shillings; 3 Primes will stand thus (.3), and that is in Value 6 Shillings. Again, a Prime or .1 being divided into ten equal Parts, each of those Parts will be one Second, and is thus expressed (.01.) and its Value will be found 2 *d.* Farthing, and $\frac{6}{10}$ of a Farthing; and so will .05 signify one Shilling or five Seconds. And if .01 be divided into ten other equal Parts, each of those Parts so divided will be Thirds, and will stand thus .001, and its Value will be found to be 96 of a Farthing, or $\frac{26}{1000}$ of a Farthing, and .009 Thirds will be 2 *d.* and .64 of a Farthing, or $\frac{1}{144}$ of a Farthing, &c. So that .375, will be found to represent 7 *s.* 6 *d.* for the 3 Primes are 6 Shillings, and the 7 Seconds are 1 *s.* 4 *d.* and $\frac{5}{10}$ of a Penny, and the 5 Thirds are 1 Penny, $\frac{8}{10}$ of a Penny, both which added together make 7 *s.* 6 *d.*

29. If you put any Bulk or Body, representing an Integer, if it be decimally divided, then the Parts in the first Decimation are Primes, the next Seconds, and the next Decimation is Thirds, the next Fourths, &c. As let there be given a Bullet of Lead, or such like, whose Weight let it be 50 *l.* Troy, this is call'd a Unit, Integer, or Thing; then will the like Weight and Matter make 10 other, the which together will be equal to 50 *l.* and will weigh each of them 5 *l.* a-piece; take of the same Matter, and equal to 5 *l.* make 10 more, then each of those weigh 6 Ounces a-piece; also, if again you take 6 Ounces, and thereof make 10 other small Bullets, each of them will weigh 12 Penny-weight Troy; and thus have you made Primes, Seconds, and Thirds, in respect of the Integer, containing 50 *l.* Troy weight; so that 5 Primes is equal to the half Mass and 2 Primes, and 5 Seconds, is a Quarter of the Mass; and therefore one of the first Division,

2 of the second Division, and 5 of the third Division, will be equal in Weight to halt a Quarter of the Mass, and contains 6 l. 3 Ounces.

30. When a Decimal Fraction followeth a whole Number, you are to separate or part the Decimal from the whole Number by a Point or Prick; so if 75 followed the whole Number 32, set them thus, 32 75. You shall find that diverse Authors, have diverse ways in expressing mixt Numbers, as thus, 32|75, or 32| $\frac{75}{100}$, or 32|75, but you will find that 32.75 thus placed and expressed, is the fittest for Calculation.

31. A mixt Number hath two Parts, the whole, and the broken; the whole is that which is composed of Integers, and the broken is a Fraction annexed thereunto. So the mixt Number $36\frac{8}{12}$ being given, we say, that 36 is the whole Number, which is composed of Integers; and the $\frac{8}{12}$ is the broken Number annexed, which sheweth that one of the former Integers (of that 36) being divided into 12 Parts, $\frac{8}{12}$, doth express 8 of those 12 Parts more, belonging to the said 36 Integers.

32. Denominative Numbers are of one, or of many, and those are of divers Sorts and Kinds, viz. Singular, called Unit, as 1; and Plural a Multitude, as 2, 3, 4, 5; Single, of one Kind only called Digits, as 1, 2, 3, 4, 5, 6, 7, 8, 9; and Compounds of many, 10, 11, 12, &c. 12, 367, &c.

Proportional, as Single, Multiple, Double, Triple, Quadruple, &c. Denominate, as Pounds, Shillings, Pence; Undenominate, as 1, 2, 3, &c. Perfect, as 6, 28, 496, 8128, 130816, 2096128, &c. whose Parts are equal to the Numbers; imperfect, unequal, and more than the Sum, as 12, to 1, 2, 3, 4, 6; imperfect, unequal and more than the Sum, as 8 to 1, 2, 4. Numbers Commensurable and Incommensurable, as 12 and 9, are Commensurable, because 3 measures them both; but 16 and 17 are Incommensurable, because no one common Number or Measure can measure them; Linear, in Form of a Line, as Superficial, in Form of a Superficies or Plane, as ::::: or ::::, &c. and Number cubical or solid, in Form of a Cube. Those two latter are otherwise called Figurative Numbers: There are also other Numbers called Tabu-

lar, as Sines, Tangents, Secants, &c. Others that be called Logarithmetick, or borrow'd Numbers, fitted to Proportion for Ease, and speedy Calculation of all manner of Questions.

C H A P. II.

Of the Natural Divisions of Integers, and the several Denominations of the Parts.

1. **A**ND that we may advance methodically herein, we will begin with the main Pillars on which Arithmetick is founded, viz. the several Species of that Art: But first,

Of Money, Weights, &c.

2. The least Denomination or Fraction of Money used in England is a Farthing, from which is produced the following Table, called the Table of Coin, viz,

				And therefore,			
				<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
1 Farth.	}	make	1 Farthing	{			
4 Farth.			1 Penny		1—20—	12—	4
12 Pence.			1 Shilling		1—20—	240—	960
20 Shil.			1 Pound			1—12—	48
				1—4			

The First of these Tables, viz. that on the left Hand, is plain and easy to be understood, and therefore wants no Direction. In the second Table above the Line, you have 1 *l.* 20 *s.* 12 *d.* 4 *qrs.* whereby is meant, That a Pound is equal to 20 Shillings, and 1 Shilling is equal to 12 Pence, and 1 Penny equal to 4 Farthings; under the Line is 1 *l.* 20 *s.* 240 *d.* 960 *qrs.* which signifies 1 *l.* to contain 20 Shillings, or 240 Pence, or 960 Farthings; in the second Line below that is 1 *s.* 12 *d.* 48 *qrs.* the First standing under the Denomination of Shillings, whereby is to be noted, that 1 Shilling is equal to 12 Pence or 48 Farthings; and likewise that below that, one Penny is equal in Value to four Farthings; understand the like Reason in all the following Tables of Weight, Measure, Time, Motion, and Dozen.

Of Troy-Weight.

3. The least Fraction or Denomination of Weight used in England, is a Grain of Wheat gathered out of the Middle of the Ear, and well dried; from whence are produced these following Tables of Weight, called *Troy-Weight*.

32 Grains of Wheat	} make {	24 Artificial Grains
24 Artificial Grains		1 Penny-weight
20 Penny-weight		1 Ounce
12 Ounces		1 Pound Troy-weight.

And therefore,

l.	oun.	p. w.	grains.
1	12	20	24
<hr/>			
1	12	240	5760
	1	20	480
		1	24

Troy Weight serveth only to weigh Bread, Gold, Silver, and E'lectuaries; it also regulateth and prescribeth a Form how to keep the Money of England at a certain Standard.

Of Apothecaries Weight.

4. The *Apothecaries* have their Weights deduced from *Troy-Weight*, a Pound *Troy* being the greatest Integer,, a Table of whose Division and Subdivision followeth, viz:

And therefore,

		l.	oun.	drams	scrup.	gr.
1 Pound	} makes {	12	Ounces	1	12	8 — 3 — 20
1 Ounce		8	Drams	1	12	96 — 288 — 5760
1 Dram		3	Scruples	1	8	24 — 480
1 Scruple		20	Grains	1	3	60
					1	20

5. Thus much concerning *Troy-Weight*, and its Derivative Weights; besides which, there is another Kind of Weight used in England, known by the Name of *Averdupois Weight*, (1 Pound of which is equal to 14 Ounces 12 Penny-weight *Troy Weight*) and it serveth to weigh all Kinds of Grocery-wares, as also Butter, Cheese, Flesh, Wax, Tallow, Rosin, Pitch, Lead, &c. the Table of which is as followeth.

A Table of Averdupois-Weight.

4 Quarters of a Dram	}	make	}	1 Dram
16 Drams				1 Ounce
16 Ounces				1 Pound
28 Pounds				1 Quarter of a Hundred
4 Quarters				1 Hund. Weight or 112 l.
20 Hundred				1 Tun

And therefore,

Tun.	C.	qr.	l.	oun.	drams.	qrs.
1	20	4	28	16	19	4
1	20	80	2240	35040	573440	2293760
	1	4	112	1792	28672	114688
		1	28	448	7168	28672
			1	16	256	1024
				1	16	64
					1	4

Wool is weighed with this Weight, but only the Divisions are not the same.

7 Pounds	}	make	}	1 Clove
2 Cloves				1 Stone
2 Stones				1 Todd
6 Todd 1 Stone				1 Wey
2 Weys				1 Sack
12 Sacks				1 Last

And therefore,

last	sack	wey	todd	stone	cloves	l.
1	12	2	6 $\frac{1}{2}$	2	2	7
1	12	24	150	312	624	4368
	1	2	13	26	52	364
		1	6 $\frac{1}{2}$	13	26	182
			1	2	4	28
				1	2	14
					1	7

Note, That in some Counties the Wey is 256 l. Averdupois, as is the Suff. l. Wey; but in Essex there is 336 l. in a Wey.

6. The least denominative Part of Liquid Measure is a Pint, which was formerly taken from *Troy weight*, (1 Pound of Wheat *Troy-weight* making a Pint of Liquid Measure) but since, by a late Act of Parliament, to prevent Frauds in the Excise, the Pint Beer Measure is to contain $35\frac{1}{4}$ solid Inches, and the Pint Wine $28\frac{7}{8}$ the like Inches, &c.

A Table of Liquid Measure.

$35\frac{1}{4}$ Cubical Inches	}	make	1 Pint Beer Measure
$28\frac{7}{8}$ Cubical Inches			1 Pint Wine Measure
2 Pints			1 Quart
2 Quarts			1 Pottle
2 Pottles			1 Gallon
8 Gallons			1 Firkin of Ale, Soap, or Beer
9 Gallons			1 Firkin of Beer
10 Gallons and half			1 Firkin of Salmon or Eels
2 Firkins			1 Kilderkin
2 Kilderkins			1 Barrel
42 Gallons	}		1 Tearce of Wine
63 Gallons			1 Hogshead
2 Hogsheads			1 Pipe or Butt
2 Pipes or Butts			1 Tun of Wine

And therefore,

<i>Tuns</i>	<i>pipes</i>	<i>hbds.</i>	<i>gall.</i>	<i>pints.</i>
1	2	2	63	8
<hr/>				
1	2	4	252	2016
	1	2	126	1008
		1	63	504
			1	8

7. The least denominative Part of dry Measure is also 1 Pint, and this is likewise taken from *Troy weight*.

A Table of Dry Measure.

1 Pound <i>Troy</i>	}	make	1 Pint
2 Pints			1 Quart
2 Quarts			1 Pottle
2 Pottles			1 Gallon

2 Gallons

2 Gilleons
4 Pecks
2 Bushels
2 Combs
2 Quarters
5 Quarters
3 Weys

} make {
1 Peck
1 Bushel
1 Comb
1 Quarter
1 Chaldron
1 Wey
1 Last.

And therefore,

last	wey	qrs.	com.	bush	pecks	gall.	pint
1	—	2	—	5	—	2	—
1	—	2	—	10	—	20	—
1	—	5	—	10	—	40	—
1	—	2	—	8	—	32	—
1	—	4	—	16	—	32	—
1	—	4	—	8	—	6	—
1	—	2	—	1	—	1	—

8. The least Denominative Part of Long-measure is Barly-corn well dried, and taken out of the Middle of the Ear, whose Table of Parts followeth.

3 Barly-corns
12 Inches
3 Feet
3 Feet ½ Inches, or a Yard and a Quarter
6 Feet
5 Yards and a Half
40 Poles or Perches
8 Furlongs

} make {
1 Inch
1 Foot
1 Yard
1 Ell English
1 Fathom
1 Pole, Perch, or Rod
1 Furlong
1 English Mile

And therefore,

mile	furl.	poles	yards	feet	inches	barlycorn
1	—	8	—	40	—	5½
1	—	8	—	320	—	1760
1	—	4	—	220	—	660
1	—	5½	—	16½	—	198
2	—	3	—	36	—	108
1	—	12	—	36	—	108

And note, that the Yard, as also the Ell, is usually divided into Quarters, and each Quarter into 4 Nails.

Note also, That a Geometrical Pace is 5 Feet, and there are 1056 such Paces in an *English* Mile.

9. The Parts of the superficial Measures of Land are such as are mentioned in the following Table, viz.

A Table of Land Measure.

40 Square Poles or Perches	} make	{ 1 Rood, or Quarter of an Acre
4 Roods		

By the foregoing Table of Long Measure, you are informed what a Pole or Perch is; and by this, that 40 square Perches is a Rood. Now a square Perch is a Superficies very aptly resembled by a square Trencher, every Side thereof being a Perch of 5 Yards and a Half in Length, 40 of them is a Rood, and 4 Roods an Acre. So that a Superficies, that is 40 Perches long, and 4 Broad, is an Acre of Land, the Acre containing in all 160 square Perches.

10. The least denominative Part of Time, is one Minute, the greatest Integer being a Year, from whence is produced this

Table of Time.

1 Minute	} make	1 Minute
60 Minutes		1 Hour
24 Hours		1 Day natural
7 Days		1 Week
4 Weeks		1 Month
13 Months, 1 day, 6 hours		1 Year

But the Year is usually divided into twelve unequal Calendar Months, whose Names, and the Number of Days they contain, are as follows, viz.

So that the Year containeth

Days	Days	365 Days, and 6 Hours; but the
<i>January</i> 31	<i>July</i> 31	6 Hours are not reckon'd but
<i>February</i> 28	<i>August</i> 31	only every fourth Year, and then
<i>March</i> 31	<i>Septemb.</i> 30	there is a Day added to the latter
<i>April</i> 30	<i>October</i> 31	End of <i>February</i> , and then it containeth
<i>May</i> 31	<i>Novemb.</i> 30	29 Days; and that Year
<i>June</i> 30	<i>Decemb.</i> 31	is called Leap-Year, and containeth 366 Days.

And here note, That as the Hour is divided into 60 Minutes, so each Minute is sub-divided into 60 Seconds, and each Second into 60 Thirds, and each Third into 60 Fourths, &c.

The Tropical Year, by the exactest Observation of the most accurate Astronomers, is found to be 365 Days, 5 Hours, 49 Minutes, 4 Seconds, and 21 Thirds.

CHAP. III.

Of the Species or Kinds of Arithmetick.

THERE are several Species of this Art ; and which may be termed either Natural, Artificial, Analytical, Algebraical, Lineal, or Instrumental : But what we are now to treat upon, relates to the single Parts of Natural Arithmetick, so far as concerns Numeration ; of which there are also four Kinds, viz. *Addition, Subtraction, Multiplication, and Division.*

CHAP. IV.

Addition of Whole Numbers.

1. **A**DDITION is the Reduction of two or more Numbers, of like Kind, together into one Sum or Total, Or it is that by which divers Numbers are added together, to the End that the Sum or Total Value of them all may be discovered.

The first Number in every Addition is called the *Addible Number* ; the other, the *Number or Numbers added* ; and the Number invented by the Addition, is called the *Aggregate or Sum*, containing the Value of the Addition.

The Collation of the Numbers, is the right placing the Numbers given respectively to each Denomination, and the Operation is the artificial adding of the Numbers given together, in order to the finding out of the Aggregate or Sum.

2. In Addition place the Numbers given respectively the one above the other, in such sort, that the like Degree, Place, or Denomination, may stand in the same Series, viz. Units under Units, Tens under Tens, Hundreds under Hundreds, &c. Pounds under Pounds, Shillings under

under Shillings, Pence under Pence, &c. Yards under Yards, Feet under Feet, &c.

3. Having thus placed the Numbers given (as before) and drawn a Line under them, add them together, beginning with the lesser Denomination, *viz.* at the right Hand; and so on, subscribing the Sum under the Line respectively: As for Example,

Let there be given 3352, and 213, and 133, to be added together. I set the Units in each particular Number under each other, and so likewise the Tens under the Tens, &c. and draw a Line under them, as in the Margent; then I begin at the Place of Units, and add them together upwards, saying, 3 and 3 are 6, and 2 makes 8, which I set under the Line, and under the same Figures added together; then I proceed to the next Place, being the Place of Tens, and add them in the same Manner as I did in the Place of Units, saying, 3 and 1 are 4, and 5 are 9, which I likewise set under the Line respectively; then I go to the Place of Hundreds, and add them up as I did the other, saying, 1 and 2 are 3, and 3 are 6, which is also set under the Line; and lastly, I go to the Place of Thousands, and because there are no other Figures to add to the 3, I set it under the Line in its respective Place, and so the Work is finished; and I find the Sum of the 3 given Numbers to be 3698.

3352

213

133

—

3698

and carry 1 (for the Ten) to the next Series; then I say, 1 that I carried and 3 are 4, and 6 are 10, now, because it comes to just 10, and no more, I set 0 under the Line, and carry 1 for the 10 to the next, and say, 1 that I carried and 2 are 3, and 3 are 6, which I set down in its respective Place; thus the Addition is ended, and the total Sum of these Numbers is found to be 6035. Several Examples of this Kind follow.

Numbers to be added		$\left\{ \begin{array}{r} 354867 \\ 573846 \\ 785946 \\ 347205 \end{array} \right.$ <hr/> Sum 2061864	
Numbers to be added	$\left\{ \begin{array}{r} 748647 \\ 455934 \\ 76483 \\ 648400 \end{array} \right.$ <hr/> Sum 1939364	Numbers to be added	$\left\{ \begin{array}{r} 45346 \\ 38074 \\ 8437 \\ 923 \\ 76 \end{array} \right.$ <hr/> Sum 92856

5. If the Numbers given to be added, are contained under divers Denominations, as of *Pounds, Shillings, Pence and Farthings*; or of *Tuns, Hundreds, Quarters, Pounds &c.* Then in this Case having disposed of the Numbers of each Denomination under other of the like Kind; beginning at the least Denomination (minding how many of one Denomination do make an Integer in the next) and having added them up, for every Integer of the next greater Denomination that you find therein contained, bear an Unit in Mind to be added to the said next greater Denomination, expressing the Excess respectively under the Line; proceed in this Manner until your Addition be finished; the following Example will make the Rule plain to the Learner. Thus these following Sums being given to be added, viz. 136 l. 13 s. 04 d. 2 qrs. and 79 l. 07 s. 10 d. 3 qrs. and 33 l. 18 s. 09 d. 1 qr. also 15 l. 09 s. 05 d. 0 qrs. The Numbers being disposed according to Order, will stand as in the Margent. Then I begin at the Denomination of Farthings, and add them up, saying,

and 3 are 4. and 2 makes 6. Now I consider that six Farthings are 1 Penny 2 Farthings; wherefore I set down the 2 Farthings in its Place under the Line; and keep 1 in Mind to be added to the next Denomination of Pence:

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
136	13	04	2
79	07	10	39
33	18	09	1
15	09	05	0

Then I go on, saying, 1 that I carried and 5 are 6, and 9 are 15, and 10 are 25, and 4 are 29; now I consider that 29 Pence are 2 Shillings and 5 Pence, therefore I set down 5 Pence in order under the Line, and keep 2 in Mind for the 2 Shillings to be added to the Shillings; then I go on saying, 2 that I carried and 9 are 11, and 18 are 29, and 7 are 36, and 13 are 49; then I consider that 49 Shillings are 2 Pounds and 9 Shillings, wherefore I set the 9 Shillings under the Line, and carry the 2 for the 2 Pounds to the next and last Denomination of Pounds; and proceed, saying, 2 that I carried and 5 makes 7, and 3 are 10, and 9 are 19, and 6 are 25; then I set down 5, and carry 2 for the 2 Tens; and proceed, saying, 2 that I carry and 1 is 3, and 3 are 6, and 7 are 13, and 3 make 16, and I set down 6, and carry 1 for the 10, and go on; saying, 1 that I carried and 1 are 2, which I set in its Place under the Line, and the Work is finished: and thus I find the Sum of the aforefaid Numbers to be 265 *l.* 9 *s.* 5 *d.* 2 *qrs.* Here is another Example in the Operation, of which the Learner must have an Eye to the Table of *Troy-Weight*. The Numbers given are 38 *l.* 7 *oz.* 13 *p. w.* 18 *gr.* and 50 *l.* 10 *oz.* 10 *p. w.* 12 *gr.* and 42 *l.* 08 *oz.* 05 *p. w.* 26 *gr.* And in order to the Addition thereof, I place them as you see, and proceed to the Operation; saying, 16 and 12 are 28, and 18 are 46; now because 24 Grains make 1 Penny-weight, 46 Grains are 1 Penny-weight, and 22 Grains, therefore I set down 22, and carry 1 for the Penny-weight, and 5 makes 6, and 10 are 16, and 13 are 29, which is one Ounce and 9 Penny-weight. I set down 9 in its Place under the Line, and carry 1 to the Ounces, saying, 1 that I carry and 8 are

265	09	05	2
-----	----	----	---

<i>l.</i>	<i>oz.</i>	<i>p.w.</i>	<i>gr.</i>
38	07	13	18
50	10	10	12
42	08	05	16

132	02	09	22
-----	----	----	----

are

are 9, and 10 are 19, and 7 are 26, and because 26 Ounces make 2 Pounds 2 Ounces, I set down 2 for the Ounces, and carry 2 to the Pounds; going on, 2 that I carry and 2 are 4, and 8 make 12, that is 2 and go 1; then 1 I carry and 4 are 5, and 5 are 10, and 3 are 13, which I set down as in the Margent, and the Work is finished, and I find the Sum of the said Numbers to amount to 132 l. 2 oz. 9 p. w. 22 gr. The Way of proving these, or any Sum in this Rule, is shewed immediately after the ensuing Example.

Addition of English Money.

l.	s.	d.	qrs.		l.	s.	d.	qrs.
436	—03	—07	—1		48	—15	—11	—1
184	—09	—10	—3		76	—10	—07	—3
768	—17	—04	—2		18	—00	—05	—3
564	—11	—01	—0		24	—19	—09	—2
<hr/>					<hr/>			
1954	—12	—09	—2		168	—06	—10	—1
<hr/>					<hr/>			

Addition of Troy-Weight.

l.	oz.	p. w.	gr.		l.	oz.	p. w.	gr.
15	—07	—13	—12		145	—09	—12	—18
18	—06	—04	—20		726	—08	—14	—10
11	—10	—16	—18		389	—07	—06	—13
09	—04	—10	—22		83	—10	—16	—20
19	—11	—18	—04		130	—00	—10	—12
22	—00	—00	—05		74	—07	—15	—03
<hr/>					<hr/>			
97	—05	—04	—09		1550	—08	—16	—01

Addition of Apothecaries Weights.

l.	oz.	dr.	sc.	gr.		l.	oz.	dr.	sc.	gr.
41	—07	—1	—0	—14		60	—03	—	—0	—10
74	—05	—5	—2	—10		48	—10	—5	—0	—14
64	—10	—7	—1	—6		34	—08	—2	—1	—15
17	—08	—1	—0	—11		18	—11	—2	—2	—11
34	—09	—6	—1	—09		160	—07	—1	—2	—15
<hr/>						<hr/>				
240	—05	—6	—1	—00		35	—2	—5	—1	—07
<hr/>						<hr/>				
						358	—07	—7	—0	—12

Addi-

Addition of Averdupois Weight.

Ton.	C.	qrs.	l.	l.	oun.	dr.
75	13	1	15	36	10	12
48	07	3	21	22	01	13
60	11	1	17	11	07	04
21	07	0	25	15	04	10
12	16	0	11	20	00	09
<hr/>				<hr/>		
218	05	0	16	106	03	00

Addition of Liquid Measure.

Tun.	pipe	hdd.	gal.	Tun	hdd.	gal.	pts.
45	1	1	48	30	3	40	4
15	0	1	17	12	0	28	6
38	0	0	47	47	5	60	5
12	1	0	56	57	3	22	3
21	1	1	18	17	0	00	0
<hr/>				<hr/>			
133	0	1	60	166	1	26	2

Addition of Dry Measure.

Chal.	qrs.	bush.	pec.	qrs.	bush	pec	gall.
48	3	7	3	17	3	1	1
13	1	4	0	50	1	3	0
54	0	6	2	14	5	3	1
16	3	6	1	40	2	0	1
4	1	0	1	30	0	3	0
<hr/>				<hr/>			
173	3	0	3	152	5	3	1

Addition of Long-Measure.

Yds.	qrs.	Nails.	Ells	qrs.	Nails.
35	3	3	56	1	3
14	1	2	13	3	2
74	2	3	48	2	1
38	0	1	50	0	2
30	1	0	74	2	0
15	0	0	17	1	0
<hr/>			<hr/>		
208	1	1	260	1	0

Addi-

Addition of Land Measure.

Acre	Rood	Perch.	Acre	Rood	Perch.
12	3	18	86	1	36
14	6	24	47	3	24
30	2	19	73	2	28
48	3	30	60	1	07
28	1	38	04	2	08
50	3	26	14	1	14
<hr/>			<hr/>		
185	3	35	286	3	37

The Proof of Addition.

6. Addition is proved after this Manner: When you have found out the Sum of the Number given, then separate the uppermost Line from the rest with a Stroke or Dash of the Pen, and then add them all up again as you did before leaving out the uppermost Line; and having so done, add the new invented Sum to the uppermost Line you separated, and if the Sum of those two Lines be equal to the Sum first found out, then the Work is performed true, otherwise not. As for Example; Let us prove the first Example of Addition of Money, whose Sum we find to be 265 l. 9 s. 5 d. 2 qrs. and which we prove thus: Having separated the uppermost Number from the rest by a Line, as you see in the Margent, then I add the same together again, leaving out the said uppermost Line, and the Sum thereof I set under the first Sum or true Sum, which doth amount to 128 l. 16 s. 1 d. 0 qrs. then again I add this new Sum to the uppermost Line that before was separated from the rest, and the Sum of those two is 265 l. 09 s. 05 d. 2 qrs. the same with the first Sum, and therefore I conclude that the Operation was rightly performed.

l.	s.	d.	qrs.
136	13	04	2
<hr/>			
79	07	10	0
33	18	09	0
15	09	05	0
<hr/>			
265	09	05	2
<hr/>			
128	16	01	0
<hr/>			
265	09	05	2

7. The main End of Addition in Question resolvable thereby, is to know the Sum of several Debris, Parcels, Integers, &c. some Questions may be these that follow.

Quest

Quest. 1. There was an old Man whose Age was required; to which he replied, I have 7 Sons, each having two Years between the Birth of each other, and in the 44th Year of my Age my eldest Son was born, which is now the Age of the youngest. I demand, What was the old Man's Age.

Now to resolve this Question, first set down the Fathers's Age at the Birth of his first Child, which was 44; then the Difference between the Oldest and the Youngest, which is 12 Years, and then the Age of the Youngest, which is 44; and then add them all together, and their Sum is 100, the compleat Age of their Father.

Quest. 2. A Man lent his Friend at several Times, these several Sums, viz. at one Time 63 *l.* at another Time 50 *l.* at another Time 48 *l.* at another Time 156 *l.* Now I desire to know how much was lent him in all?

Set the Sums lent one under another, as you see in the Margent, and then add them together, and you will find their Sum to amount to 317 *l.* which is the Total of all the several Sums lent, and so much is due to the Creditor.

317

Quest. 3. There are two Numbers, the least whereof is 40, and their Difference 14. I desire to know what is the greater Number, and also what is the Sum of them both? First set down the least, viz. 40 and 14, the Difference, and add them together, and their Sum is 54 for the greatest Number; then I set 40 (the least) under 54 (the greatest) and add them together; and their Sum is 94, equal to the greatest and least Numbers.

40

14

—

54

40

—

Sum 94

C H A P. V.

Of Subtraction of whole Numbers.

Subtraction, is taking of a lesser Number out of a greater of a like Kind, whereby to find out a third Number, being or declaring the Inequality, Excess or Difference between the Numbers given; or *Subtraction* is that by which one Number is taken out of another Number given, to the End that the Residue or Remainder may be known, which Remainder is also called the Rest, Remainder, or Difference of the Numbers given.

2. The Number out of which *Subtraction* is to be made must be greater, or at least equal with the other Number given; the higher Number is called the *Major*, and the lower, *Minor*; and the Operation of *Subtraction* being finished, the Rest or Remainder is called the *Difference* of the Numbers given.

3. In *Subtraction*, place the Numbers given respectively, the one under the other, in such Sort as like Degrees, Places, or Denominations may stand in the same Series, viz. Units under Units, Tens under Tens, Pounds under Pounds, &c. Feet under Feet, and Parts under Parts, &c. This being done, draw a Line underneath, as in *Addition*.

4. Having placed the Numbers given as is before directed, and draw a Line under them, subtract the lower Number (which in this Case must always be less than the Uppermost) out of the higher Number, and subscribe the Difference or Remainder respectively below the Line, and when the Work is finished, the Number below the Line, will give you the Remainder.

As for Example, Let 364521 be given to be subtracted from 795836, I set the lesser under the greater, as in the Margin, and draw a Line under them; then beginning at the right Hand, I say, one out of 6, and there remains 5, which I set in order under the Line; then I proceed to the next, saying, 2 from 3 rests 1, which I note also under the Line; and thus I go on till I have finished the

795836
364521
—
431315
Work;

Work ; and then I find the Remainder or Difference to be 43815.

5. But if it so happen (as commonly it doth) that the lowermost Number or Figure is greater than the uppermost, then in this Case add ten to the uppermost Number, and subtract the said lowermost Number from their Sum, and the Remainder place under the Line, and when you go to the next Figure below, pay an Unit, by adding it thereto for the Ten you borrowed before, and subtract that from the higher Number of Figures, and thus go on till your Subtraction be finished. As for Example, Let 437503 be given, from whence it is required to subtract 153827, I dispose of the Numbers as is before directed, and as you see in the Margent ; then I begin, saying, 7 from 3 I cannot, but (adding 10 thereto) I say, 7 from 13 and there remains 6, which I set under the Line in order ; then I proceed to the next Figure, saying, 1 that I borrowed and 2 is 3 from 0 I cannot, but 3 from 10 and there remain 7, which I likewise set down as before ; then 1 that I borrowed and 8 is 9, from 5 I cannot, but 9 from 15, and there remain 6 ; then 1 I borrowed and 3 is 4. from 7, and there remain 3 ; then 5 from 3 I cannot, but 5 from 13, and there remain 8 ; then 1 borrowed and 1 are 2, from 4, and there rest 2 ; and thus the Work is finished : After these Numbers are subtracted one from another, the Inequality, Remainder, Excess, or Difference, is found to be 283676. Examples for thy farther Experience may be these that follow.

From 3469916
Take 738642

Rest 2731274

From 361977
T-k: 5864

Rest 355713

6. If the Sum or Number to be subtracted is of several Denominations, place the lesser Sum below the greater, and in the same Rank and Order, as is shewed in Addition of the same Numbers ; then begin at the right Hand, and take the lower Number out of the uppermost, if it be lesser ; but if it be bigger than the Uppermost, then borrow an Unit from the next greater Denomination, and

turn it into the Parts of the less Denomination, and add those Parts to the Uppermost, noting the Remainder below the Line; then proceed and pay one to the next Denomination for that which you borrowed before, and proceed in this Order till the Work be finished. An Example of this Rule followeth: Let 375 *l.* 13 *s.* 7 *d.* 1 *qr.* be given from whence let it be required to subtract 57 *l.* 16 *s.* 03 *d.* 2 *qrs.* In order whereunto, I place the Numbers as you see in the Margent; and thus I begin at the least Denomination, saying 2 from 1 I cannot, therefore I borrow one Penny from the next Denomination, and turn it into Farthings, which is 4, and adding 4 to 1, which is 5, I say, but 2 from 5, and there remain 3, which I put under the Line; then going on, I say, 1 that I borrow'd and 3 is 4 from 7, and there rests 3; then going on, I say, 16 from 13 I cannot, but borrowing 1 Pound and turning it into 20 Shillings, I add to it 13, and that is (33) wherefore I say, 16 from 33 and there remains 17, which I set under the Line, and go on; saying 1 that I borrowed and 7 is 8, from 5 I cannot, but 8 from 13 and there remain 5; and the 1 that I borrowed and 5 is 6 from 7 there rests 1, and 0 from 3 rests 3, and the Work is done. And I find the Remainder or Difference to be 317 *l.* 17 *s.* 3 *d.* 3 *qrs.*

Another Example of *Troy-weight*, may be this. I would subtract 17 *l.* 10 *oz.* 11 *p.w.* 20 *gr.* from 24 *l.* 5 *oz.* 00 *p.w.* 08 *gr.* I place the Numbers according to the Rule, and begin, saying, 20 from 8 I cannot, but I borrow 1 Penny-weight, which is 24 Grains and add them to 8, and there are 32, wherefore I say 20 from 32 rests 12; then 1 that I borrowed and 11 is 12, from 00 I cannot, but 12 from 20 (borrowing an Ounce, which is 20 Penny-weight) and there remain 8, then 1 that I borrowed and 10 is 11, from 5 I cannot, but 11 from 17, and there rests 6; then 1 that I borrowed, and 7 is 8, from 4 I cannot, but 8 from 13

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
375	13	07	
57	16	03	
<hr/>			
317	17	03	

<i>l.</i>	<i>oz.</i>	<i>p.w.</i>	<i>gr.</i>
24	05	00	
17	10	11	
<hr/>			
06	06	08	

and there rest 6 ; then 1 that I borrowed and 1 is 2, from 2, and there rests nothing ; so that I find the Remainder or Difference to be 6 l. 6 oz. 8 p. w. 12 gr.

7. It many times happeneth that you have many Sums or Numbers to be subtracted from one Number, as suppose a Man should lend his Friend a certain Sum of Money, and his Friend hath paid him Part of his Debt at several times, then before you can conveniently know what is still owing, you are to add the several Numbers or Sums of Payment together, and subtract their Sum from the whole Debt, and the Remainder is the Sum due to the Creditor ; as suppose A lendeth to B 564 l. 16 s. 10 d. and B hath repaid him 79 l. 16 s.

8 d. at one time, and 163 l.

18 s. 11 d. at another time,

and 241 l. 15 s. 8 d. at another time ; and you would

know how the Account

standeth between 'em, or what

is more due to A. In order

that whereunto I first set down the

Sum which A lent, and draw

a Line underneath it, then un-

derneath that Line I set the several

Sums of Payment, as you see in the Margent ; and having

brought the several Sums of Payment into one Total by

the 5th Rule of the fourth Chapter foregoing, I find their

Sum amounteth to 485 l. 11 s. 3 d. which I subtract

from the Sum first lent by A, by the 6th Rule of this

Chapter, and I find the Remainder to be 79 l. 5 s. 7 d.

and so much is still due to A.

When the Learner hath good Knowledge of what hath

been already delivered in this and the foregoing Chapters,

he will with Ease understand the Manner of working the

following Examp^ls.

Subtraction of whole Numbers.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Borrowed	374	10	03
Paid	79	15	11
Remain	294	14	04

C 2

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
	700	10	11	2
	9	03	11	3
	691	06	11	3
				Borrowed

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Lent	564	16	10
Paid at several Payments	79	16	08
	163	18	11
	241	15	08
Paid in all	485	11	03
Remain	79	05	07

	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Borrowed	1000	00	00		711	03	00	0
Paid	19	00	06		11	13	00	1
Remain	680	19	06		699	09	11	3

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Borrowed	3300	00	00	0

Paid at several Payments	170	10	00	0
	361	13	10	1
	590	03	04	3
	73	04	11	3

Paid in all	1195	12	02	3
-------------	------	----	----	---

Remain due	2104	07	09	1
------------	------	----	----	---

Subtraction of Troy-weight.

	<i>l.</i>	<i>oz.</i>	<i>p w.</i>	<i>gr.</i>
Bought	174	00	13	00
Sold	78	04	16	15

Remain	95	07	16	09
--------	----	----	----	----

	<i>l.</i>	<i>oz.</i>	<i>p w.</i>	<i>gr.</i>
Bought	470	10	13	00

Sold at several Times	60	00	00	00
	35	10	18	00
	16	07	09	08
	48	04	00	00
	61	11	19	23
	23	00	00	00

Sold in all	245	10	07	07
-------------	-----	----	----	----

Remain unfold	225	00	05	17
---------------	-----	----	----	----

Subtraction of Apothecaries Weight.

	<i>l.</i>	<i>oz.</i>	<i>dr.</i>	<i>sc.</i>	<i>gr.</i>		<i>l.</i>	<i>oz.</i>	<i>dr.</i>	<i>sc.</i>	<i>gr.</i>
Bought	12	04	3	0	00		20	00	1	0	07
Sold	8	05	1	1	15		10	00	1	2	12
Rem.	03	11	1	1	05		09	11	7	0	15

Subtraction

Subtraction of Averdupois-weight.

	C.	qrs.	l.		tu.	C.	qrs.	l.	oz.	dr.
Bought	35	0	15		5	07	1	10	10	05
Sold	16	2	20		3	17	1	16	09	13
	<hr/>				<hr/>					
Remain	18	1	23		1	09	3	22	00	08

Subtraction of Liquid Measure.

	tu.	hhd.	gal.		tu.	hhd.	gal.	pints.
Bought	40	1	30		60	3	42	4
Sold	16	1	40		15	3	46	6
	<hr/>				<hr/>			
Remain	23	3	53		44	3	58	6

Subtraction of Dry Measure.

	chal.	qrs.	bufh.	pec.		chal.	qrs.	bufh.	pec.
Bought	100	0	0	0		73	2	3	2
Sold	54	1	4	3		46	2	3	3
	<hr/>					<hr/>			
Remain	45	2	3	1		26	3	7	3

Subtraction of Long Measure.

	yds.	qrs.	nails		yds.	qrs.	nails
Bought	160	0	0		344	0	1
Sold	64	1	2		177	1	3
	<hr/>				<hr/>		
Remain	95	2	2		106	2	2

Subtraction of Land Measure.

	acres	rood	perch.		acres	rood	perch.
Bought	140	2	13		600	0	00
Sold	70	3	12		54	0	16
	<hr/>				<hr/>		
Remain	69	3	01		545	5	24

The Proof of Subtraction.

8. When your Subtraction is ended, if you desire to prove the Work, whether it be true or no; then add the Remainder to the minor Number, and if the Aggregate of these two be equal to the major Number, then is your Operation true, otherwise false: Thus let us prove the first Example of the fifth Rule of this Chapter, where after Subtraction is ended, the Numbers stand as in the Margin,

the Remainder or Difference being 283676. Now to prove the Work, I add the said Remainder 283976 to the minor Number 153827, by the fourth Rule of the foregoing Chapter, and I find the Sum or Aggregate to be 437503, equal to the major Number, or Number from whence the lesser is subtracted. Behold the Work in the Margent.

The Proof of another Example, may be of the first Example of the 6th Rule of this Chapter, where it is required to subtract 57 l. 16 s. 3 d. 2 qrs. from 375 l. 13 s. 7 d. 1 qr. and by the Rule I find the Remainder to be 317 l. 17 s. 03 d. 3 qrs. Now to prove it, I add the said Remainder 317 l. 17 s. 03 d. 3 qrs. to the minor Number 57 l. 16 s. 03 d. 2 qrs. and their Sum is 375 l. 13 s. 7 d. 1 qr. equal to the major Number, which proves the Work to be true; but if it had happened to have been either more or less than the said major Number, then the Operation had been false.

9. The general Effect of Subtraction, is, to find the Difference or Excess between two Numbers, and the Rest when a Payment is made in Part of a greater Sum, the Date of Books printed, the Age of any Thing, by knowing the present Year, and the Year wherein they are made, created, or built, and such like.

The Questions appropriated to this Rule, are such as follow.

Quest. What Difference is there between one Thing of 125 Foot long, and another of 66 Foot long?

To resolve this Question, I first set down the major or greater Number 125, and under it the minor or lesser Number 66, as is directed in the third Rule of this Chapter, and according to the fourth Rule of the same, I subtract the Minor from the Major, and the Remainder, Excess, or Difference I find to be 59. See the Work in the Margent.

Quest. 2. A Gentleman hath owed a Merchant 365 l. whereof he hath paid 278 l. What more doth he owe?

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To give an Answer to this Question, I first set down the major Number 365 *l.* and under it I place 278 the minor, and subtract the one from the other, whereby I discover the Excess, Difference, or Remainder, to be 87; and so much is still due to the Creditor, as per Margent.

Quest. 3. An Obligation was written, a Book printed, a Child born, a Church built, or any other Thing made in the Year of our Lord 1572, and now we account the Year of our Lord 1687, the Question is to know the Age of the said Things; that is, How many Years are passed since the said Things were made? I say, if you subtract the lesser Number 1572, from the greater 1687, the Remainder will be 115, and so many Years are passed since the Making of the said Things; as by the Work in the Margent.

Quest. 4. There are three Towns lie in a straight Line, viz. London, Huntingdon, and York, now the Distance between the farthest of these Towns, viz. London and York, is 151 Miles, and from London to Huntingdon is 49 Miles. I demand how far it is from Huntingdon to York?

To resolve this Question, subtract 49 the Distance between London and Huntingdon, from 151, the Distance between London and York, and the Remainder is 102 for the true Distance between Huntingdon and York. See the Work in the Margent.

C H A P. VI.

Of Multiplication of Whole Numbers.

1. **M**ultiplication is performed by two Numbers of like Kind for the Production of a Third, which shall have such Reason to the one, as the other hath to the Unit, and in Effect is a most brief and artificial Compound Addition of many equal Numbers of like Kind into one Sum. Or, Multiplication is that by which we multiply two or more Numbers, the one into the other, to the End that their Product may come forth, or be discovered.

Or, *Multiplication* is the Increasing of any one Number by any other, so often as there are Units in that Number, by which the other is increased; or by having two Numbers given to find a third, which shall contain one of the Numbers as many times as there are Units in the other

2. *Multiplication* hath three Parts. First, The Multiplicand or Number to be multiplied. Secondly, The Multiplier or Number given by which the Multiplicand is to be multiplied. And thirdly, The Product or Number produced by the other two, the one being multiplied 8 by the other; as if 8 were given to be multiplied by 4, I say 4 times 8 is 32; here 8 is the Multipli- 8
cand, and 4 is the Multiplier. and 32 is the Pro- 32
duct.

3. *Multiplication* is either *Single*, by one Figure; or *Compound*, that consists of many.

Single Multiplication is said to consist of one Figure, because the Multiplicand and Multiplier consist each of them of a Digit, and no more; so that the greatest *Product* that can arise by *Single Multiplication* is 81, being the Square of 9; and *Compound Multiplication* is said to consist of many Figures, because the Multiplicand or Multiplier consists of more Places than one; as if I were to multiply 436 by 6: It is called *Compound*, because the Multiplicand 436 is of more Places than one, viz. 3 Places.

4. The Learner ought to have all the Varieties of *Single Multiplication* by Heart, before he can well proceed any farther in this Art, it being of most excellent Use, and none of the following Rules in *Arithmetick*, but what have a principal Dependence thereupon, which may be learnt by the following Table.

Multiplication TABLE.

1.	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The Use of the precedent Table is this : In the uppermost Line or Column you have expressed all the Digits from 1 to 9 ; and likewise beginning at 1 and going downwards in the side Column, you have the same : so that if you would know the Product of any two single Numbers multiplied by one another, look for one of them (which you please) in the uppermost Column, and for the other in the side Column, and running your Eye from each Figure along the respective Columns in the common Angle (or Place) where these two Columns meet, there is the Product required. As for Example, I would know how much is 8 times 7 ; First I look for 8 in the uppermost Column, and 7 in the side Column ; then do I cast my Eye from 8 along the Column downwards from the same, and likewise from 7 in the side Column, I cast my Eye from thence toward the right Hand, and find it to meet with the first Column at 56, so that I conclude 56 to be the Product required, &c.

5. In *Compound Multiplication*, if the Multiplicand consists of many Places, and the Multiplier of but one Figure ; first set down the Multiplicand, and under it place the Multiplier in the Place of Units, and draw a Line underneath them ; begin then and multiply the Multiplier into every particular Figure of the Multiplicand, beginning at the Place of Units, and so proceed towards the left Hand, setting each particular Product under the Line, in order as you proceed : But if any of the Products exceed

ceeds 10, or any Number of Tens, set down the Excess and for every 10 carry an Unit to be added to the next Product, always remembering to set down the total Product of the last Figure; which Work being finished, the Sum or Number placed under the Line shall be the true and total Product required. As for Example, I would multiply 478 by 6: First set down 478, and underneath it 6, in the Place of Units, and draw a Line underneath them, as in the Margent; then I begin, saying, 6 times 8 is 48, which is 8 above four Tens, therefore I set down 8 (the Excess) and bear 4 in Mind for the 4 Tens; then I proceed, saying, 6 times 7 is 42, and that I carried is 46, I then set down 6 and carry 4, and go on, saying, 6 times 4 is 24, and 4 that I carried is 28, and because it is the last Figure, I set it all down, and so the Work is finished, and the Product is found to be 2868, as was required.

6. When in *Compound Multiplication*, the Multiplier consisteth of divers Places, then begin with the Figure in the Place of Units in the Multiplier, and multiply it into all the Figures in the Multiplicand, placing the Product below the Line, as was directed in the last Example; then begin with the Figure of the second Place of the Multiplier, viz. the Place of Tens, and multiply it likewise into the whole Multiplicand, (as you did the first Figure) placing its Product under the Product of the first Figure; do in the same Manner by the Third, Fourth, and Fifth, &c. until you have multiplied all the Figures of the Multiplier particularly into the whole Multiplicand, still placing the Product of each particular Figure under the Product of its precedent Figure; herein observing the following Caution.

In the placing of the Product of each particular Figure of the Multiplier, you are not to follow the 2d Rule of the 4th Chapter, viz. to place Units under Units, and Tens under Tens, &c. but to place the Figure or Cypher in the Place of Units of the second Line under the second Figure or Place of Tens in the Line above it, and the Figure or Cypher in the Place of Units in the third Line under the Place of Tens in the second Line, &c. observing this Order, till

A Caution.

you

you have finished the Work, still placing the first Figure of every Line or Product under the second Figure or Place of Tens in that which was above it, and having so done, draw a Line under all these particular Products, and add them together; so shall the Sum of all these Products be the total Product required.

As if it were required to multiply 764 by 27, I set 'em down the one under the other, with a Line drawn underneath them; then I begin, saying, 7 times 4 is 28, then I set down 8 and carry 2; then I say, 7 times 6 is 42, and 2 that I carried is 44, that is 4 and go 764
4; then 7 times 7 is 49, and 4 that I carry is 53, which I set down, because I have not another Figure to multiply; thus I have done with 27
—
the 7, then I begin with the 2, saying 2 times 4 is 8, which I set down under (4) the second Figure or Place of Tens in the Line above it, as you 5348
1528
—
may see in the Margent; then I proceed, saying, 2 times 6 is 12, that is 2 and carry 1; then 2 times 7 is 14, and 1 that I carry is 15, which I set down, because 'tis the Product of the last Figure; so that the Product of 764 by 7 is 5348, and by 2 is 1528, which being placed the one under the other, as before directed, as you see in the Margent, and a Line drawn under them, and they added together respectively, make 20628, the true Product required, being equal to 27 times 764. 20628

Another Example may be this; Let it be required to multiply 5486 by 455, I dispose of the Multiplicand and Multiplier according to the Rule, and begin multiplying the first Figure of the Multiplier, which is (5) into the whole Multiplicand, and find the Product is 27430; then I proceed, and multiply the second Figure (6) of the Multiplier into the Multiplicand, and find the Product to amount 32916, which is subscribed under the other Product respectively; then do I multiply the third and last Figure (4) of the Multiplier into the Multiplicand, and the Product is 21944, which is likewise placed under the second Line respectively; then I draw a Line under the said Products being placed the one under the other

$$\begin{array}{r}
 5486 \\
 455 \\
 \hline
 27430 \\
 32916 \\
 21944 \\
 \hline
 2550990
 \end{array}$$

the other according to Rule) and add them together and the Sum is 2550990, the true Product sought, being equal to 5486 times 465, or 465 times 5486.

More Examples in this Rule are these following.

$$\begin{array}{r}
 430865 \\
 4739 \\
 \hline
 3877185 \\
 12,2595 \\
 3016055 \\
 1723460 \\
 \hline
 2041869235
 \end{array}$$

$$\begin{array}{r}
 6400758 \\
 37496 \\
 \hline
 38404548 \\
 57606822 \\
 25603032 \\
 44805306 \\
 19202274 \\
 \hline
 240002821968
 \end{array}$$

Compendium in Multiplication.

7. Although the former Rules are sufficient for all Cases in *Multiplication*, yet because in the Work of *Multiplication* many times great Labour may be saved, I shall acquaint the Learner with some *Compendiums* in order thereto, viz. If the *Multiplicand* or *Multiplier*, or both of them, end with *Cyphers* then in your mul-

Si numeris propositis unus vel uterque adjunctus habeat ad dextram circules, omissis circulis fiat ipsorum numerorum multiplicatio, et facto demum tot insuper integrorum loci accenseantur quot sunt omissi circuli in utroque factore. Clavis Mat. c. 4. 3.

tiplying you may neglect the *Cyphers*, and multiply only the significant *Figures*, and to the *Product* of those significant *Figures*, add so many *Cyphers* as the *Numbers* given to be multiplied did end with; that is, annex them on the right Hand of the said *Product*, so shall that give you the true *Product* required. As if I were to multiply 32000 by 4300, I set them down in order to be multiplied, as you see in the *Margent*; but neglecting the *Cyphers* in both *Numbers*, I only multiply 32 by 43, and the *Product* I find to be 1376, to which I annex the 5 *Cyphers* in the *Multiplicand* and *Multiplier*, and

$$\begin{array}{r}
 32000 \\
 4300 \\
 \hline
 96 \\
 128 \\
 \hline
 137600000
 \end{array}$$

and then it makes 13760000 for the true Product of 30000 by 4300.

8. If in the Multiplier, Cyphers are placed between significant Figures, then multiply only by the significant Figures neglecting the Cyphers; but here special Notice is to be taken of the true placing of the first Figure after the Neglect of such Cypher or Cyphers; and therefore you must observe in what Place of the Multiplier the Figure you multiply by standeth, and set the first Figure of that Product under the same Place of the Product of the first Figure of your Multiplier: As for Example, Let it be required to multiply 371568 by 40007.

Si intermedio multiplicantis loco circulus fuerit, ille negligitur. Alsted. c.6. De Arithm.

First I multiply the Multiplicand by 7, and the Product is 2600976; then neglecting the Cyphers, I multiply by 4, and that Product is 1486272; now I consider, that 4 is the 5th Figure in the Multiplier, therefore I place 2 (the first Figure of the Product by 4) under the fifth Place of the first Produce by 7, and the rest in order, and having added them together, the total Product is found to be 14865320976. Other Examples in this Rule, are these following:

371568
40007

2600976
1486272
14865320976

327586
6030

7864371
20604

9827580
1965516

31457484
47186226
15723712

1975343516

162037500084

9. If you are to multiply any Number by an Unit with Cyphers, as by 10, 100, 1000, &c. then annex so many Cyphers before the Multiplicand, and that Number when the Cyphers are annex'd, is the Product requir'd. As if you would multiply 428 by 100, annex 2 Cyphers to 428, and it is 42800. If it were required to multiply 102 by 10000,

10000

10000, annex 4 Cyphers, and it gives 1020000 for the Product required.

The Proof of Multiplication.

10. *Multiplication* is proved by *Division*, and to speak Truth, all other Ways are false, (according to *Frisius*) and therefore it will be necessary in the first Place to learn *Division*, and by that to prove *Multiplication*. There are some other Ways used indeed, but on a strict *Examen*, there is not one in a Thousand of their Products right; therefore we omit them.

11. The general Effect of *Multiplication* is contained in the Definition of the same, which is to find out a third Number, so often containing one of the two given Numbers, as the other containeth Units.

The second Effect is, by having the Length and Breadth of any Thing (as a Parallelogram or long Plain) to find the superficial Content of the same, and by having the superficial Content of the Base, and the Length, to find out the Solidity of any Parallelopipedon, Cylinder, or other solid Figures.

The third Effect is, by the Contents, Price, Value, Buying, Selling, Expence, Wages, Exchange, Simple Interest, Gain or Loss of any one thing, be it Money, Merchandize, &c. to find out the Value, Price, Expence, Buying, Selling, Exchange, or Interest of any Number of Things of like Name, Nature, and Kind.

The fourth Effect (is not much unlike the other) by the Contents, Value, or Price of any one Part of any Thing denominated, to find the Contents, Value, or Price of the whole Thing, all the Parts into which the whole is divided, multiplying the Price of one of those Parts.

The fifth Effect is, to aid, to compound, and to make other Rules, as chiefly, the *Rule of Proportion*, called the *Golden Rule*, or *Rule of Three*; also by it, Things of one Denomination are reduced to another.

If you multiply any Number of Integers, or the Price of the Integer, the Product will discover the Price of the Quantity, or Number of Integers given.

In a Rectangular Solid, if you multiply the Breadth of the Base by the Depth, and that Produce by the Length, the last Product will discover the Solidity or Content of the same Solid.

Some Questions proper to this Rule, may be these following :

Quest. 1. What is the Content of a square Piece of Ground, whose Length is 28 Perches, and Breadth 13?

Answer, 364 square Perches; for multiplying 28 the Length, by 13 the Breadth, the Product is so much.

Quest. 2. There is a square Battle, whose Flank is 47 Men, and the Files 19 deep, what Number of Men doth that Battle contain? *Facit* 893; for multiplying 47 by 19, the Product is 893.

Quest. 3. If any one Thing cost 4 Shillings, what shall 9 Things cost? *Ans.* 36 Shillings; for multiplying 4 by 9, the Product is 36.

Quest. 4. If a Piece of Money or Merchandize be worth, or cost 17 Shillings, what shall 19 such Pieces of Money or Merchandize cost? *Facit.* 323 Shillings, which is equal to 16 l. 3 s.

Quest. 5. If a Soldier or Servant get or spend 14 s. per Month, what is the Wages or Charges of 49 Soldiers or Servants for the same Time? Multiply 49 by 14, the Product is 686 s. or 14 l. 6 s. for the answer.

Quest. 6. If in a Day there are 24 Hours, how many Hours are there in a Year, accounting 365 Days to constitute the Year? *Facit.* 8760 Hours; to which if you add the 6 Hours over and above 365 Days, as there is in a Year, then it will be 8766 Hours; now if you multiply this 8766 by 60, the Number of Minutes in an Hour, it will produce 525960, the Number of Minutes in a Year.

C H A P. VII.

Division of whole Numbers.

1. **D**ivision, is the separating or parting of any Number or Quantity given, into any Parts assigned, or to find how often one Number is contained in another; or from any two Numbers given, to find a third that shall consist of so many Units, as the one of those two Numbers given is comprehended or contained in the other.

2. *Division* hath three Parts or Numbers remarkable, viz. First, The Dividend; 2dly, The Divisor; 3dly,

The Quotient. The Dividend is the Number given to be parted or divided. The Divisor is the Number given by which the Dividend is divided, or it is the Number which sheweth how many Parts the Dividend is to be divided into. And the Quotient is the Number produced by the Division of the two given Numbers the one by the other.

So 12 being given to be divided by 3, or into three equal Parts, the Quotient will be 4; for 3 is contained in 12 four times, where 12 is the Dividend, and 3 is the Divisor, and 4 is the Quotient.

3. In Division set down your Dividend, and draw a crooked Line at each End of it, and before the Line at the left Hand place the Divisor, and behind that on the right Hand place the Figures of the Quotient, as in the Margent, where it is required to divide 12 3) 12 (4 by 3: First, I set down 12 the Dividend, and on each Side of it, do I draw a crooked Line, and before that on the left Hand do I place 3 the Divisor; then do I seek how often 3 is contained in 12; and because I find it four times, I put 4 behind the crooked Line, on the right Hand of the Dividend, denoting the Quotient.

4. But if, when the Divisor is a single Figure, the Dividend consisteth of two or more Places, then having placed them for the Work (as before directed) but a Point under the first Figure of the left Hand of the Dividend, provided it be bigger than (or equal to) the Divisor; but if it be lesser than the Divisor, then put a Point under the second Figure from the left Hand of the Dividend; which Figures, as far as the Point goeth from the left Hand, are to be reckon'd by themselves, as if they had no Dependence upon the other Part of the Dividend: And for Distinction sake may be called the Dividual; then ask how often the Divisor is contained in the Dividual; placing the Answer in the Quotient; then multiply the Divisor by the Figure that you placed in the Quotient, and set the Product thereof under your Dividual; then draw a Line under the Product, and subtract the said Product from the Dividual, placing the Remainder under the said Line; then put a Point under the next Figure in the Dividend on the right Hand of that to which you put the Point before, and draw

draw it down, placing it on the right Hand of the Remainder which you found by *Subtraction*; which Remainder, with the said Figure annexed before it, shall be a new Dividual; then see again how often the Divisor is contained in this new Dividual, and put the Answer in the Quotient on the right Hand of the Figure which you put there before; then multiply the Divisor by the last Figure that you put in the Quotient, and subscribe the Product under the Dividual, and make *Subtraction*, and to the Remainder draw down the next Figure from the grand Dividend, (having first put a Point under it) and put it on the right Hand of the Remainder for a new Dividual as before, and proceed thus till the Work is finished.

Observing this general Rule in all Kinds of *Division*. First, to seek how often the Divisor is contained in the Dividual; then (having put the Answer in the Quotient) multiply the Divisor thereby, and subtract the Product from the Dividual. An Example or two will make the Rule plain. Let it be required to divide 2184 by 6. I dispose of the Numbers given as is before directed, and as you see in the Margent; in order to the

Work, then because 6 the Divisor is more (6) 2184 (3
than 2 the first Figure of the Dividend) I :
put a Point under 1 the second Figure,
which makes the 21 for the Dividual, then do I ask how
often 6 the Divisor is contained in 21, and because I can-
not have it more than 3 times, I put 3
in the Quotient, and thereby do I multiply (6) 2184 (3)
the Divisor (6) and the Product is 18,
which I set in order under the Dividual,
and subtract it therefrom, and the Re-
mainder (3) I place in order under the
Line) as you see in the Margent.

Then do I make a Point under the next
Figure of the Dividend, being 8, and
draw it down, placing it before the Re-
mainder 3, so have I 38 for a new Divi-
dual, then do I seek how often 6 is con-
tained in 38, and because I can't have it
more than 6 Times, I put 6 in the Quo-
tient, and thereby do I multiply the Divisor 6, and the

$$\begin{array}{r}
 6) 2184 \quad (3 \\
 \underline{18} \\
 3 \\
 6) 2184 \quad (36 \\
 \underline{18} \\
 38 \\
 \underline{36} \\
 2
 \end{array}$$

Pro-

Product (36) I put under the Dividual (38) and subtract it therefrom, and the Remainder 2 I put under the Line as you see in the Margent.

Then do I put a Point under the next (and last) Figure of the Dividend (being 4) and draw it down to the Remainder 2, and putting it on the eight Hand thereof, it maketh 24 for a new Dividual; when I ask how often 6 is contained in 24, and the Answer is 4. which I put in the Quotient, and multiply the Divisor (6) thereby, and the Product (24) I put under the Dividual (24) and subtract it therefrom, and the Remainder is (0); and thus the Work is finished, and I find the Quotient to be 364, that is 6 is contained in 2184, just 364 times, or 2184 being divided into 6 equal Parts, 364 is one of those Parts.

$$\begin{array}{r}
 6) 2184 \quad (364 \\
 \dots \\
 18 \\
 \hline
 38 \\
 36 \\
 \hline
 24 \\
 24 \\
 \hline
 0
 \end{array}$$

Again, If it were required to divide 2646 by 7, or into 7 equal Parts, the Quotient will be found to be 378, by the following Operation appeareth.

$$\begin{array}{r}
 7) 2646 \quad (378 \\
 \dots \\
 21 \\
 \hline
 34 \\
 49 \\
 \hline
 56 \\
 56 \\
 \hline
 00
 \end{array}$$

So if it be required to divide 946 by 8, the Quotient will be found to be 118, and 2 remaining after Division ended. The Work followeth :

$$8) 946$$

$$\begin{array}{r}
 8 \overline{) 946} \quad (118 \\
 \underline{8} \\
 14 \\
 \underline{8} \\
 66 \\
 \underline{54} \\
 (2)
 \end{array}$$

Many times the Dividend cannot exactly be divided by the Divisor, but something will remain, as in the last Example, where 946 was given to be divided by 6, the Quotient was 118, and there remained 2 after the Division was ended: Now what is to be done in this Case with the Remainder, the Learner shall be taught when we come to treat of the Reducing (or Reduction) of Fractions.

And here note, That if after your Division is ended, any Thing do remain, it must be lesser than your Divisor; for otherwise your Work is not rightly performed.

Other Examples are such as follow.

$$\begin{array}{r}
 8 \overline{) 73464} \quad (9183 \\
 \underline{0} \\
 72 \\
 \underline{0} \\
 14 \\
 \underline{8} \\
 66 \\
 \underline{64} \\
 24 \\
 \underline{24} \\
 (0)
 \end{array}$$

$$\begin{array}{r}
 9 \overline{) 13758} \quad (1528 \\
 \underline{0} \\
 9 \\
 \underline{0} \\
 47 \\
 \underline{45} \\
 25 \\
 \underline{18} \\
 78 \\
 \underline{72} \\
 (6)
 \end{array}$$

5. But if the Divisor consisteth of more Places than one, then chuse so many Figures from the left Side of the Dividend for a Dividual as there are Figures in the Divisor, and put a Point under the farthest Figure of that Dividual

to the right Hand, and seek how often the first Figure on the left Side of the Divisor is contained in the first Figure on the left Side of the Dividual, and place the Answer in the Quotient, and thereby multiply your Divisor, placing your Product under your Dividual, and subtract it therefrom, placing the Remainder below the Line; then put a Point under the next Figure in the Dividend, and draw a Line down to the said Remainder, and annex it on the right Side thereof, which makes a new Dividual, and proceed as before, till the Work is finished.

And if it so happen, that after you have chosen your first Dividual, (as it is before directed) you find it to be lesser than the Divisor, then put a Point under the Figure more nearly to the right Hand, and seek how often the first Figure on the left Side of the Divisor is contained in the two first Figures on the left Side of the Dividual, and place the Answer in the Quotient, by which multiply the Divisor, and place the Product thereof in order under the Dividual, and subtract it therefrom, and proceed as before.

Always remembering, that in all Cases of Division, after you have multiplied your Divisor by the Figure first placed in the Quotient, the Product be greater than the Dividual, then you must cancel that Figure in the Quotient, and instead thereof put a Figure lesser by an Unit (or One) and multiply the Divisor thereby, and if still the Product be greater than the Dividual, make the Figure in the Quotient yet lesser by an Unit, and thus do until your Product be lesser than the Dividual, or at the most equal thereto, and then make Subtraction, &c.

So if you would divide 9464 by 24, the Quotient will be found to be 394; I first put down the given Number as is before directed in the 3d Rule. Now because my Divisor consisteth of two Figures, I therefore put a Point under the second Figure from the left Hand of my Dividend, which there is 4, wherefore I seek how often 2 the first Figure (on the left Side of the Divisor) is contained in 9 (the like first in the Dividual) the Answer is 4, which I put in the Quotient, and thereby multiply all the Divisor, and find the Product to be 96, which is greater

$$\begin{array}{r}
 24 \ 9464 \ (394 \\
 \dots \\
 72 \\
 \hline
 226 \\
 216 \\
 \hline
 10
 \end{array}$$

than the Dividual 94, wherefore I cancel the 4 in the Quotient, and instead thereof I put 3, (an Unit lesser) and by it multiply the Divisor 24, and the Product is 72, which I subtract from 94 the Dividual, and the Remainder is 22; then do I make a Point under the next Figure 6 in the Dividend, and draw it down, and place it on the right Side of the Remainder 22, and it makes 226 for a new Dividual; now because the Dividual 226 consisteth of a Figure more than the Divisor, therefore I seek how often 2 (the 1st Figure of the Divisor) is contained in 22, the two first of the Dividual, and I say 9 times, wherefore I put 9 in the Quotient, and thereby multiply the Divisor 24, the Product (216) I place under the Dividual 226, and subtract it from it, and there remaineth 10.

$$\begin{array}{r}
 24) 9464 \text{ (39)} \\
 \dots \\
 72 \\
 \hline
 226 \\
 216 \\
 \hline
 10
 \end{array}$$

Then I go on and make a Point under the next and last Figure (4) in the Dividend, and draw it down to the Remainder 10, and it makes 104 for a new Dividual, which is also a Figure more than the Divisor; and therefore I seek how often 2 is contained in 10, I answer 5 times; but multiplying my Divisor by 5, the Product is 120, which is greater than the Dividual, and therefore I make it but 4, and by it multiply the Divisor, and the Product is 96, which being placed under, and subtracted from the Dividual, there remaineth 8; and thus the whole Work of this Division is ended, and I find that 9464, being divided by 24, or into 24 equal Parts, is found to be 394, as was said before; and the Remainder is 8, as you see in the Work following.

$$\begin{array}{r}
 24) 9464 \text{ (394)} \\
 \dots \\
 72 \\
 \hline
 226 \\
 216 \\
 \hline
 104 \\
 96 \\
 \hline
 (8)
 \end{array}$$

Another

Another Example may be this : Let there be required the Quotient of 183653 divided by 385 ; First I dispose of the Numbers in order to their dividing, and because 118, the three first Figures of the Dividend, is lesser than the Divisor 385, and therefore make a Point under the fourth Figure, which is 3, and see how often 3 (the first Figure of the Divisor) is contained in 11 : The Answer is 3, which I put in the Quotient, and thereby multiply the Divisor 385, and the Product is 1155, which I subtract from the Dividend 183, and there remains 28. Then (as before) I draw down the next Figure, which is 6, and place it before the Remainder 28 ; so have I 268 for a new Dividual, and because it hath no more Figures than the Divisor, I seek how often 3 (the first Figure of the Divisor) is contained in 2 (the first Figure of the Dividual) and the Answer is 0 ; for a greater Number cannot be contained in a Lesser ; wherefore I put 0 in the Quotient, and thereby (according to the 5th Rule) I should multiply the Divisor ; but if I do, the Product will be 0, and 0 subtracted from the Dividual 286, the Remainder is the same ; wherefore I draw down the next Figure (5) from the Dividend, and put it before the said Remainder 286, so have I 2865 for a new Dividual ; and because it consisteth of 4 Places, viz. a Place more than the Divisor, I seek how often 3, the 1st Figure of the Divisor, is contained in 28, the two first of the Dividual, and I say, there is 9 times 3 in 28 ; but multiplying my whole Divisor (385) thereby, I find the Product to be 3465, which is greater than the Dividual 2865 ; wherefore I chuse 8, which is lesser by an Unit than 9, and thereby I multiply my Divisor 385, and the Product is 3080, which still is greater than the said Dividual ;

$$\begin{array}{r}
 385 \overline{) 1183653} \\
 \underline{1155} \\
 28
 \end{array}$$

$$\begin{array}{r}
 385 \overline{) 1183653} \quad (30 \\
 \underline{1155} \\
 286
 \end{array}$$

$$\begin{array}{r}
 385 \overline{) 1183653} \quad (307 \\
 \underline{} \\
 1155 \\
 \underline{2865} \\
 2695 \\
 \underline{} \\
 170
 \end{array}$$

al; wherefore I chuse another Number yet an Unit lesser,
 viz 7, and have multiplied my Divisor thereby, the
 Product is 2695, which is lesser than the Dividual 2865,
 wherefore I put 7 in the Quotient and subtract 2695 from
 the Dividual 2865, and there remain 170; then I draw
 down the last Figure (3) in the Dividend, and place it
 before the said Remainder 170, and it makes 1703 for a
 new Dividual; then (for the Reason above said) I seek how often 3
 is contained in 17. the Answer is 5, but multiplying the Divisor
 thereby, the Product is 1925, greater than the Dividual, where-
 fore I say it will bear 4 (an Unit lesser) and by it I multiply the
 Divisor 385. and the Product is 1540, which is lesser than the
 Dividual, and therefore I put 4 in the Quotient, and subtract the
 said Product from the Dividual, and there remain 163; and thus the Work is finished; and
 I find that 1183653 being divided by 385, or into 385
 equal Shares or Parts, the Quotient, (or one of those Parts)
 is 3074, and besides there is 163 remaining.

$$\begin{array}{r}
 385 \overline{) 1183653} \quad (3074 \\
 \underline{1155} \\
 2865 \\
 \underline{2695} \\
 1703 \\
 \underline{1540} \\
 163
 \end{array}$$

And thus the Learner being well vers'd in the Method
 of the foregoing Examples, he may be sufficiently quali-
 fied for the dividing of any greater Sum or Number into as
 many Parts as he pleaseth, that is, he may understand the
 Method of dividing by a Divisor, which consisteth of 4,
 or 5, or 6, or any greater Number of Places, the Me-
 thod being the same with the foregoing Example in every
 Respect.

Other Examples in Division.
 27986) 835684790 (29860

55972

275964

251874

40907

223888

170199

167916

Remain (22830)

196374) 473986018 (2413

392748

812380

785496

268841

196374

714678

589122

Remain (15556)

So if you divide 47386473 by 58736, you will find the Quotient to be 806, and 45257 will remain after the Work is ended.

In like manner, If you would divide 3846739204 by 483064, the Quotient will be 7963, and the Remainder after Division will be 100572.

Compendium in Division.

1. **I**F any given Number be to be divided by another Number that hath Cyphers annexed on the right Side thereof, (omitting the Cyphers) you may cut off

Chap. 7. Whole Numbers.

many Figures from the right Hand of the Dividend, as there are Cyphers before the Divisor, and let the remaining Numbers in the Dividend, be divided by the remaining Number, or Numbers of the Divisor, observing this Caution: That if after your Division is ended, any Thing remain, you are to annex thereto the Number or Numbers that were cut off from the Dividend; and such new found Number shall be the Remainder. (See Mr. Oughtred's *Clavis Mathematica*, cap. 5. 3.) As for Example, Let it be required to divide 46658 by 400, now because there are two Cyphers before the Divisor, I cut off as many Figures from before the Dividend, viz. 58, so that then there will remain only 466 to be divided by 4, and the Quotient will be 116, and there will remain 2, to which I annex the two Figures (58) which were cut off from the Dividend, and it makes 258 from the true Remainder; so that I conclude 46658 being divided by 400, the Quotient will be 116, and 258 remain after the Work is ended; as by the Work in the Margent.

$$\begin{array}{r}
 4 \overline{) 466} 58 \quad (116 \\
 \underline{00} \\
 4 \\
 \underline{00} \\
 6 \\
 \underline{00} \\
 26 \\
 24 \\
 \underline{00} \\
 (258)
 \end{array}$$

2. And hence it followeth, that if the Divisor be 1, or an Unit with Cyphers annexed, you may cut off so many Figures from before the Dividend, as there are Cyphers in the Divisor, and then the Figure or Figures that are on the left Hand will be the Quotient, and those that are on the right Hand will be the Remainder after the Division is ended. (Vid. *Gem. Fris. Arith. Par. 1.*) As thus; if 45783 were to be divided by 10, I cut off the last Figure (3) with a Dash thus, 4578|3. and the Work is done, and the Quotient is 4578 (the Number on the left Hand of the Dash) and the Remainder is 3 (on the right Hand.) In like manner if the same Number 45783 were to be divided by 100, I cut off two Figures from the End thus, (457|83) and the Quotient is 457, and the Remainder is 83. And if I am to divide the same Figures, by 1000, I cut off 3 from the End thus (45|783) and the Quotient is 45, and 783 is the Remainder, &c.

D 6. The

6. The general Effect of *Division*, is contained in the Definition of the same, that is, by having two unequal Numbers, to find a third Number in such Proportion to the Dividend, as the Divisor hath to Unit or 1 : It also discovers what Reason or Proportion there is between Numbers; so if you divide 12 by 4, it quotes 3, which shews the Reason or Proportion of 4 to 12 is triple.

The second Effect is, by the superficial Measure or Content, and the Length of any Oblong, Rectangular Parallelogram, or square Plane known, to find out the Breadth thereby; or contrarywise, by having the Superficies and Breadth of the said Figure, to find out the Length thereof. Also by having the Solidity and Length of a Solid, to find the Superficies of the Base, *&c. contra*.

The third Effect is, by the Contents, Reason, Price, Value, Buying, Selling, Expences, Wages, Exchange, Interest, Profit, or Loss of any Number of Things (be it Money, Merchandize, or what else) to find out the Contents, Reason, Price, Value, Buying, Selling, Expence, Wages, Exchange, Interest, Profit, or Loss, or any one Thing of the like Kind.

The fourth Effect is, to aid, to compose, and to make other Rules, but principally the Rule of *Proportion*, called the *Golden Rule*, or *Rule of Three*, and the Reduction of Monies, Weights, and Measures of one Denomination into another; by it also Fractions are abbreviated, by finding a common Mesurer unto the Numerator and Denominator, thereby discovering commensurable Numbers.

If you divide the Value of any certain Quantity by the same Quantity, the Quotient discovers the Rate or Value of the Integer; as if 8 Yards of Cloth cost 20 Shillings, if you divide (96) the Value or Price of the given Quantity, by (8) the same Quantity, the Quotient will be 128, which is the Price or Value of 1 of those Yards, *&c.*

If you divide the Value or Price of any unknown Quantity, by the Value of the Integer, it gives you in the Quotient that unknown Quantity, whose Price is thus divided; as if 12 Shillings were the Value of a Yard, I would know how many Yards are worth 96 Shillings: Here if you divide (96), the Price or Value of the unknown Quantity, by 12, the Rate of the Integer, or 1 Yard, the Quotient will be 8, which is the Number of Yards worth 96 s. *Some*

Some Questions answered by Division may be these following.

Quest. 1. If 22 Things cost 66 Shillings, what will 1 such Thing cost ! *Facit* 3 Shillings, for if you divide 66 by 22, the Quotient is 3 for the Answer ; so if 26 Yards or Ells of any Thing be bought or sold for 108 l. how much will one Yard or Ell be bought or sold for ? *Facit* 3 l. for if you divide 108 by 26 Yards, the Quotient will be 3 l. the Price of the Integer.

Quest. 2. If the Expence, Charges, or Wages of 7 Years, amount to 868 l. what is the Expence, Charges, or Wages of one Year ? *Facit* 124 l. for if you divide 868 (the Wages of 7 Years) by 7 (the Numbers of Years) the Quotient will be 124 l. for the Answer. See the Work :

$$7) 868 (124$$

...

7

16

14

28

23

(0)

Quest. 3. If the Content of one superficial Foot be 144 Inches, and the Breadth of a Board be 9 Inches, how many Inches of that Board in Length will make such a Foot ? *Facit* 16 Inches ; for by dividing 144 (the Number of square Inches in a square Foot) by 9, (the Inches in the Breadth of the Board) the Quotient is 16 for the Number of Inches in Length of that Board to make a superficial Foot.

$$9) 144 (16 \text{ Inches}$$

9

54

54

(0)

D 2

Quest.

Quest. 4 If the Content of an Acre of Ground be 160 Square Perches, and the Length of a Furlong (propounded) be 80 Perches, how many Perches will there go in Breadth to make an Acre : *Facit* 2 Perches ; for if you divide 160, the Number of Perches in an Acre, by 80, (the Length of the Furlong in Perches) the Quotient is 2 Perches ; and so many in Breadth of that Furlong will make an Acre.

80) 160 (2 Perches.

..

160

(0)

Quest. 5. If there be 893 Men to be made up into a Battle, the Front consisting of 47 Men ; what Number must there be in the File ? *Facit*, 19 deep in the File ; for if you divide 893 (the Number of Men) by 47 the Number in the Front) the Quotient will be 19 in Depth of the File. The Work followeth.

47) 893 (19 Deep in File.

..

47

423

423

(0)

Quest. 6. There is a Table whose superficial Contents is 72 Feet, and the Breadth of it at the End is 3 Feet ; now I demand what is the Length of this Table ? 3) 72 (24
Facit 24 Feet long ; for if you divide 72 (the Content of the Table in Feet) by 3 (the Breadth of it (the Quotient is 24 Feet for the Length thereof, which was required. See the Operation in the Margent.

..

6

12

21

(0)

The Proof of Multiplication and Division.

Multiplication and Division interchangeably prove each other ; for if you would prove a Sum in Division, whether the Operation be right or no, multiply the Quotient

by the Divisor ; and if any Thing remain after Division is ended, add it to the Product, which Product (if your Sum was rightly divided) will be equal to the Dividend. And contrariwise, if you would prove a Sum in Multiplication, divide the Product by the Multiplier, and if the Work was rightly performed, the Quotient will be equal to the Multiplicand. See the Example, where the Work is done and undone. Let 7654 be given to be multiplied by 3242, the Product will be 24814268, as by the Work appeareth.

$$\begin{array}{r}
 7654 \\
 3242 \\
 \hline
 15308 \\
 30616 \\
 15308 \\
 22962 \\
 \hline
 24814268
 \end{array}$$

And then if you divide the said Product 24814268 by 3242 the Multiplier, the Quotient will be 7654, equal to the given Multiplicand.

$$3242) 24814268 \quad (7654$$

$$\begin{array}{r}
 \dots \\
 22694 \\
 \hline
 21202 \\
 19452 \\
 \hline
 17506 \\
 16210 \\
 \hline
 12968 \\
 12968 \\
 \hline
 (0)
 \end{array}$$

In like Manner (to prove a Sum or Number in Division) if 24814268 were divided by 3242, the Quotient will be found to be 7654 ; then for Proof. if you multiply 7654 the Quotient, by 3242 the Divisor, the Product will amount to 24814268, equal to the Dividend.

Or you may prove the last, or any other Example in Multiplication, thus, viz. Divide the Product by the Multiplicand, and the Quotient will be equal to the Multiplier. See the Work.

$$\begin{array}{r}
 7654 \\
 3232 \\
 \hline
 15308 \\
 33616 \\
 15308 \\
 22962 \\
 7654) 24814268 \quad (3242 \\
 \hline
 \dots \\
 22962 \\
 \hline
 18522 \\
 15308 \\
 \hline
 32146 \\
 30616 \\
 \hline
 15338 \\
 15308 \\
 \hline
 \end{array}$$

(o)

From whence there arises this Corollary, that any Operation in *Division*, may be proved by *Division*; for if after your *Division* is ended, you divide the Dividend by the Quotient, the new Quotient thence arising will be equal to the *Divisor* of the first Operation; for Tryal whereof, let the last Example be again repeated.

$$3242) 24814268 \quad (7654$$

$$\begin{array}{r}
 27694 \\
 \hline
 21202 \\
 19452 \\
 \hline
 17506 \\
 16210 \\
 \hline
 12968 \\
 12968 \\
 \hline
 \end{array}$$

(o)

Nor

For Proof whereof divide again 24814268 by the Quotient 7654, and the Quotient hence will be equal to the Divisor 3244 See the Work :

$$\begin{array}{r}
 7654 \overline{) 24814268} \quad (3242 \\
 \underline{22962} \\
 18522 \\
 \underline{15308} \\
 32146 \\
 \underline{30616} \\
 15308 \\
 \underline{15308} \\
 (0)
 \end{array}$$

But in proving *Division by Division*, the Learner is to Observe this following Caution : That if after his Division is ended, there be any Remainder, before you go about to prove your Work, subtract the Remainder out of your Dividend, and then work, as in the following Example, where it is required to divide 43876 by 765, the Quotient here is 57, and the Remainder is 271. See the Work following.

$$\begin{array}{r}
 765 \overline{) 43876} \quad (57 \\
 \underline{3825} \\
 5622 \\
 \underline{5355} \\
 (271)
 \end{array}$$

Now to prove this Work, subtract the Remainder 271 out of the Dividend 43876, and there remaineth 43605, for a new Dividend to be divided by the former Quotient 57, and the Quotient thence arising is 765, equal to the given Divisor, which proveth the Operation to be right.

$$\begin{array}{r}
 43876 \\
 271 \\
 \hline
 57) 43605 \text{ (765)} \\
 \quad \dots \\
 \quad 399 \\
 \hline
 \quad 370 \\
 \quad 342 \\
 \hline
 \quad 285 \\
 \quad 285 \\
 \hline
 \quad (0)
 \end{array}$$

Thus have we gone through the four Species of *Arithmetick*, viz. *Addition*, *Subtraction*, *Multiplication*, and *Division*, upon which all the following Rules, and all other Operations whatsoever that are possible to be wrought by Numbers, have their immediate Dependance, and by them are resolved. (Vide *Gem. Fris. Arith. Part 1.*) Therefore before the Learner make a farther Step in this Art, let him be well acquainted with what has been delivered in the foregoing Chapters.

C H A P. VIII.

Of Reduction.

1. **R**eduction is that which brings together two or more Numbers of different Denominations into one Denomination, [*Hall's Arith. c. 13. p. 152.*] or it serveth to change or alter Numbers, Money, Weight, Measure of Time; from one Denomination to another; and likewise to abridge Fractions to the lowest Terms. All which it doth so precisely, that the first Proportion remaineth without the least Jot of Error or Wrong committed; so that it belongeth as well to the Fractions as Integers of which in the proper Place. Reduction is generally performed either by *Multiplication* or *Division*; from whence we may gather, That,

2. Reduction is either Ascending or Descending.

3. Red

3. *Reduction Descending*, is when it is required to reduce a Sum or Number of a greater Denomination, into a Lesser; which Number, when it is so reduced, shall be equal in Value to the Number first given in the greater Denomination; [*Wing. Arith* 7, 2, 3, 4.] as if it were required to know how many Shillings, Pence, or Farthings, are equal in Value to 100 *l*. Or how many Ounces are contained in 4500 Weight. Or how many Days, Hours, or Minutes, there are in 240 Years, &c. And this Kind of Reduction is generally performed by Multiplication.

4. *Reduction Ascending*, is when it is required to reduce or bring a Sum or Number of a smaller Denomination into a greater, which shall be equivalent to the given Number; as suppose it were required to find out how many Pounds, Shillings, or Pence, are equal in Value to 43785 Farthings: Or, how many Hundreds are equal to (or in) 3748 Pounds, &c. and this Kind of Reduction is always performed by Division.

5. When any Sum or Number is given to be reduced into another Denomination, you are to consider whether it ought to be resolved by the Rule Descending or Ascending, &c. by Multiplication or Division; if it be to be performed by Multiplication, consider how many Parts of the Denomination into which you would reduce it, are contain'd in an Unit or Integer of the given Number, and multiply the said given Number thereby, and the Product thereof will be the Answer to the Question. As if the Question were in 38 Pounds how many Shillings? Here I consider, that in one Pound are 20 Shillings, and that the Number of Shillings in 38 Pounds will be 20 times 38, wherefore I multiply 38 *l*. by 20, and the Product is 760, and so many Shillings are contained in 38 Pounds, as in the Margent.

But when there is a Denomination or Denominations between the Number given and the Number required, you may (if you please) reduce it into the next interior Denomination, and then into the next lower than that, &c. until you have brought it into the Denomination required. As for Example, Let it be demanded in 132 Pounds, how many Farthings? First, I multiply 132 (the Number of Pounds given) by 20. to bring it into Shillings,

and it makes 2640 Shillings, then do I multiply the Shillings 2640 by 12, to bring them into Pence, and it produceth 31680, and so many Pence are contained in 2640 Shillings, or 132 Pounds; then do I multiply the Pence, viz. 31680 by 4, to bring them into Farthings (because 4 Farthings is a Penny) and I find the Product thereof to be 126720, and so many Farthings are equal in Value to 132 Pounds. As by the Work in the Margin.

$$\begin{array}{r}
 132 \text{ Pounds} \\
 20 \\
 \hline
 2640 \text{ Shill.} \\
 12 \\
 \hline
 31680 \\
 2640 \\
 \hline
 31680 \text{ Pence} \\
 4 \\
 \hline
 126720 \text{ Farth.}
 \end{array}$$

6. And if the Number propounded to be reduced is to be divided, or wrought by the Rule ascending, consider how many of the given Numbers are equal to an Unit, or Integer in that Denomination to which you would reduce your given Number, and make that your Divisor, and the given Number your Dividend; and the Quotient thence arising will be the Number sought or required: As for Example, let it be required to reduce 2640 Shillings into Pounds. Here I consider that 20 Shillings are equal to one Pound; wherefore I divide 2640 (the given Number) by 20, and the Quotient is 132, and so many Pounds are contained in 2640 Shillings. In Reduction descending and ascending, the Learner is advised to take particular Notice of the Tables delivered in the second Chapter of this Book, where he may be informed what Multipliers and Divisors to make Use of in the reducing of any Number to any other Denomination whatsoever, especially English Money, Weights, Measures, Time, and Motion; but in this Place it is not convenient to meddle with Foreign Coins, Weights or Measures.

$$\begin{array}{r}
 20 \overline{) 2640} (132 \\
 \underline{40} \\
 240 \\
 \underline{200} \\
 400 \\
 \underline{400} \\
 0
 \end{array}$$

But if in Reduction ascending, it happen that there is a Denomination or Denominations between the Number given and the Number required, then you may reduce your

Number

Number given into the next superiour Denomination, and when it is so reduced, bring it into the next above that, and so on until you have brought it into the Denomination required. As for Example, Let it be demanded in 126720 Farthings how many Pounds? First I divide my given Number, being Farthings, by 4, to bring them into Pence, (because 4 Farthings make one Penny) and there are 31680 Pence; then I divide 31680 Pence by 12, and the Quotient giveth 2640 Shillings, and then I divide 2640 Shillings by 20, and the Quotient giveth 132 Pounds, which are equal in Value to 126720 Farthings: See the whole Work as it followeth.

	12)	210)	l.
4) 126720	(31680	(2640	(132
.....	
12	24	2	
6	75	6	
4	72	6	
27	48	4	
24	48	4	
32	(0)	(0)	
32			
(0)			

7. When the Number given to be reduced consisteth of divers Denominations, as Pounds, Shillings, Pence, and Farthings, or of Hundreds, Quarters, Pounds, and Ounces, &c. then you are to reduce the highest (or greatest) Denomination into the next inferiour, and add thereunto the Number standing in the Denomination, which your greatest or highest Number is reduced to; then reduce that Sum into the next inferiour Denomination; adding thereto the Number standing in that Denomination; do so

l.	s.	d.
48	13	10
20		
960 Shill.		
Add	13	
Sum	973	Shill.
	12	
	1946	
	973	
	11676 Pence	
Add	10	
Sum	11686 Pence	
	11686 Pence	

More Examples in Reduction of Coin.

Quest. 1. In 438 l. how many Shillings?

438 l.

Facit. 8760 Shillings; for by multiplying the 438 by 20, the Product amounteth to so much. See the Work in the Margent.

20

Facit 8760 s.

Quest. 2. In 467 l. how many Pence? First multiply the given Number of Pounds (467) by 20, to bring it into Shillings, and it makes 9340 Shillings, then multiply the Shillings by 12, and it produceth 112080 Pence, as in the Margent.

467 Pounds

20

9340 Shill.

12

112080

9340

Fac. 112080 Pence

Or it may be resolved thus, viz. multiply the given Number of Pounds (467) by (240) the Number of Pence in a Pound, and the Product is the same, viz 112080 Pence, as by the Operation appeareth.

467 Poun.

240

112080

934

Facit 112080 Pence

Quest. 3. In 5673 l. how many Farthings? First multiply the given Number by 20, to bring it into Shillings, and it produceth 113460 Shillings, then multiply that Product by 12, to bring it into Pence, and it produceth 1361520 Pence; then lastly multiply the Pence by 4, and it produceth 5446080 Farthings.

5673 Pounds

20

113460 Shillings

12

226920

113460

1361520 Pence

4

Facit 5446080 Farth.

Or this Question might have been thus resolved, viz. multiply 5673 the given Number of the Pounds by 960 (the Number of Farthings in a Pound) and it produceth the same Effect, as you may see by the Work.

5673 Pounds	20 Shillings.
960	12
<hr/>	<hr/>
340380	240 Pence
51057	4
<hr/>	<hr/>

Facit 5446080 Farthings

960 Farthings

Otherwise thus : First bring the given Number 5673 l. into Shillings, and multiply the Shillings by 48. the Number of Farthings in a Shilling, and the same Effect is thereby likewise produced, viz.

5673 Pounds	12 Pence
20	4
<hr/>	<hr/>
113460 Shillings	48 Farthings.
48	
<hr/>	
907680	
453840	
<hr/>	

Facit 5446080 Farthings.

These various Ways of Operation are expressed to inform the Judgment of the Learner, with the Reason of the Rule. More Ways may be shewn, but these are sufficient even for the meanest Capacities.

Quest. 4. In 458 l. 16 s. 7 d. 3 qrs. how many Farthings? To resolve this Question, consider the 7th Rule of this Chapter, and work as you are there directed, and you find the aforesaid given Numbers to amount to 440479 Farthings, viz.

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
458	16	7	3
20			

Add 9:60
16 *Shillings.*

Sum 9176 *Shillings.*
12

12,52
9176

110112 *Pence.*

Add 7

Sum 110119 *Pence.*
4

440476 *Farthings.*

Add 3

Sum 440479 *Farthings.*

This last Question, or any other of this kind, may be more concisely resolved thus, *viz.* When you multiply the Pounds by 20, to bring them into Shillings, to the Product of the first Figure, add the Figure standing in the Place of Units in the Denomination of Shillings; but because the first Figure in the Multiplier is (0) I say, 0 times 8 is nothing, but 6 is 6, which I put down for the first Figure in the Product, then because the Multiplier is 0, I go on no further with it; for if I should, the whole Product would be 0, but proceed; and when I come to multiply by the second Figure in the Multiplier, to the Product of it, I add the Figure standing in the Place of Tens in the Denominations of Shillings, which is 1, saying, 2 times 8 is 16, and (the said Figure) 1 is 17; then I set down 7, and carry the Unit to the Product of the next Figure, as is directed in the 5th Rule of the 6th Chapter foregoing, and finish the Work. So that now you may have the whole

Product

Product and Sum of Shillings at one Operation, which is the same as before; and when you multiply the Shillings by 12, to bring them into Pence, (after the same Manner) add to the Product the Number standing in the Denomination of Pence, and to when you multiply the Pence by 4, to bring them into Farthings, add to the Product the Number standing under the Denomination of Farthings. See the last Question thus wrought.

l.	s.	d.	qrs.
458	16	7	3
20			
<hr/>			
9176	Shillings		
12			
<hr/>			
18359			
9176			
<hr/>			
110119			
4			
<hr/>			

Facit 440479 Farthings.

After the Method last prescribed, are all the following Examples, that are of the same Nature, wrought and resolved.

Quest. 5. In 4375866 Farthings, I demand how many Pounds, Shillings, Pence, and Farthings?

To resolve this Question, First, I divide the given Number of Farthings by 4, and the Quotient is 1093966 Pence, and there remaineth 2 after the Division is ended, which (by the 8th Rule foregoing) is two Farthings; then I divide 1093966 Pence by 12, and the Quotient is 91163 Shillings, and there remaineth 10 after Division, which by the said 8th Rule is so many Pence, viz. 10 d. then I divide 91163 Shillings by 20, and the Quotient 4558 l. and there remaineth 3 Shilling; so the Work is finished, and I find that in 4375866 Farthings, here are 4558 l. 3 s. 10 d. 2 qrs. See the Operation.

4) 4375866	(12)	(2) 0	l.
.....	(1093966	(9116 3	(4558
.....
4	108	8	
37	13	11	
36	12	10	
15	19	11	
12	12	10	
38	76	16	
36	72	16	
26	46	(3) s.	
24	36		
26	(10) d.		
24			

(2) qrs. l. s. d. qrs.
Facit 4558 3 10 2

Quest. 6. In 4386 l. I demand how many Groats?

To resolve this Question, I reduce the given Number of Pounds into Shillings, and they are 87720 Shillings, now I consider that in a Shilling are 3 Groats, therefore I multiply the Shillings by 3, and it produceth 263160 Groats See the Work.

4386 Pounds
20
87720 Shillings
3
Facit 263160 Groats.

This Question might have been otherwise resolved thus, viz. considering that in a Pound (or 20 Shillings) there are three Times 20 Groats, which makes 60, by which I multiply the Number of Pounds given, and it produceth the same Effect at one Operation, as followeth.

4386 Pounds.

60 Groats in 20 s.

Facit 263160 Groats 4386 l.

Quest. 7. In 43758 Three-pences, I desire to know how many Pounds?

To resolve this, and many such like Questions; First, I divide my given Number of Three-pences by 4, because 4 Three-pences are in a Shilling, and the Quotient is 10939 Shillings, and there remaineth 2 after Division ended, which is two Three-pences (by the 8th Rule of the Chapter) which are equal in Value to 6 *d.*, then I divide 10939 Shillings by 20, and the Quotient giveth 546 l. and 19 s. remains: So that I conclude in 43758 Pieces of Three-pence *per Piece*, there are 546 l. 19 s. 6 d. as by the Work appeareth:

4) 43758	2 0	l.	s.	d.
....	(1093 9	(546	19	06
4			
<hr/> 37	<hr/> 10			
36	9			
<hr/> 15	8			
12	<hr/> 13			
<hr/> 38	12			
36	<hr/> 19			
<hr/>				

19 Shillings.

(2) Three-pences, or 6 d.

This Question might have been otherwise resolved thus *viz.* First multiply the given Number of Three-pence 43758, by 3 the Number of Pence in Three-pence, and the Product (*viz.* 131274) is the Number of Pence equal to the given Number of Three-pences, which Number of Pence may be brought into Pounds by dividing by 12, and by 20, and the Quotient you will find to be equal to the former Work, 546 l. 19 s. 6 d.

$$\begin{array}{r}
 43758 \\
 \hline
 3 \overline{) 210} \\
 \hline
 12 \overline{) 131274} \quad (1093 \overline{) 9} \quad (546 \overline{) 19} \quad 6.
 \end{array}$$

12	10
112	9
108	8
47	13
36	12
114 re. (19) Shillings	
108	
(6) Pence remains.	

Or thus, Divide, the given Number of 3 Pences by the Number of 3 Pences in a Pound, or 20 Shillings (which you will find to be 80, if you multiply 20 s. by 4, the Number of 3 Pences in a Shilling) and you will find the Quote to be 546 l. as before, and a Remainder of 78 Three-pences; and if you divide those 78 Three-pences by 4 (because there are 4 Three-pences in a Shilling, you will find the Quote to be 19 s. and 2 Three-pences remain, which are equal to 6 d. which is the same that was before found.

$$8|0) 4375|8 \quad (546 \quad 19 \quad 6$$

20

4

40

80

37

31

55

48

4) 78 (19 s.

4

38

36

(2) Three-pences or 6 d.

Quest. 8. In 4785 l. 13 s. how many Pieces of 13 d. per Piece?

This Question cannot be resolved by Reduction Descending or Ascending absolutely, (because 13 d. $\frac{1}{2}$ is no Part of a Pound) but rather by them both jointly, viz. Multiplication and Division; for if you bring the Number given into Half-pence, and divide the Half-pence by Half-pence in 13 d. $\frac{1}{2}$, viz. 27, the Quotient will be the Answer; for having brought 4785 l. 13 s. into Half-pence, I find it makes 2297112, which I divide by 27 (because there are so many Half-pence in 13 d. $\frac{1}{2}$) the Quote gives 85078 Pieces of 3 d. $\frac{1}{2}$, and 6 Half-pence remain over and above: Observe the Work following.

l. s.
4785 13.
20

d.
13 $\frac{1}{2}$

95713 Shillings
24 Half-pence in a Shilling.

2
27 Half-pence

382852
191426

2297112 Half-pence in the given Number
27) 2297112 (85078 Pieces of 13 d. $\frac{1}{2}$.

216

137
135

211
189

222
216

Remains (6) Half-pence.

It would have produced the same Answer, if you had reduced your given Number into Farthings, and divided by the Farthings in 13 d. $\frac{1}{2}$ viz. 54; (for always the Dividend and the Divisor must be of one Denomination) and then you would have had a Remainder of 12 Farthings, which are equal in Value to the former Remainder of 6 Half-pence, as you may prove at your Leisure.

Quest. 9. In 540 Dollars at 4 s. 4 d. per Dollar. how many Pounds Sterling?

First, bring your given Number of Dollars into Pence, and then your Pence into Pounds, according to the former Directions, Thus in 4 s. 4 d. (viz. a Dollar, you will find 52 Pence, by which multiply 540 Dollars, and it produceth 28080 Pence, which if you divide by 246 (the Pence in one Pound) the Quotient will give you 117 l. which are equal in Value to 540 Dollars, at 4 s. 6 d. per Dollar.

540

	s.	d.
540	4	4
51	12	
<hr/>		
1080	52	
2700		
<hr/>		
240) 28080 (117 l.		
...		
24		
<hr/>		
40		
24		
<hr/>		
168		
168		
<hr/>		
(0)		

The foregoing Question might have been otherwise wrought thus, viz. Multiply 540, your Given Number of Dollars, by 13 the Number of Groats in a Dollar; or 4s. 4d. and it produceth 7020 Groats, which divide by 60, the Groats in one Pound, or 20 Shillings, and the Quote is 117, as before. See the Work.

	s.	d.
540	4	
133	3	
<hr/>		
1620	13	
540		
<hr/>		
60) 7020 (117		
...		
6		
<hr/>		
10		
6		
<hr/>		
42		
42		
<hr/>		
(0)		

Quest. 10 In 547386 Pieces of 4 d. $\frac{1}{2}$ per Piece, how many Pounds, Shillings, and Pence?

First bring your given Number Four-pence Half-pence all into Half-pence, which you will do if you multiply by 9, the Number of Half-pence in 4 d. $\frac{1}{2}$, and the Product is 4926474 Half-pence, which are brought into Pounds

if you divide them by 24, the Half-pence in a Shilling, and 20, the Shillings in a Pound, 'it makes 10263 l. 9 s. 9 d.

547386	9		d.		
24) 4926474	210	l.	4 $\frac{1}{2}$	2	
.....			9 Half-pence.	
48	2				
126	05				
120	4				
64	12				
48	12				
167	6				
144	6				
234 rem.	(9) Shillings.				
216					

Rem. (18 Half-pence, or 9 d.

Quest. 11. In 4386 l. I demand how many Pieces of d. of 4 d. and of 2 d. of each an equal Number? That is to say, What Number of Six-pences, Groats, and Two-pences will make 4386 l. and the Number of each equal?

The Way to resolve Questions of this Nature, is to add several Pieces, into which the given Number is to be brought into one Sum, and reduce the given Number into the same Denomination with their Sum, and to divide the said given Number (so reduced) by the said Sum, and the Quotient will give you the exact Number of each Piece. And after the same Method will we proceed to solve the present Question, viz.

4386 Pounds
240 Pence

6 d.
4 d.
2

175440
8772

Sum 12 d.

12) 1052640 (87720

.....

96

92

84

86

84

24

24

(0)

Facit 87720 Pieces of 6 d.

So that I conclude by the Operation, that 87720 Six-pences, and 87720 Groats, and 87720 Two-pences, is just as much as (or equal to) 4386 l. or if you admit 5 s. to be thus divided, it is equal to 5 Six-pences, and 5 Four-pences or Groats, and 5 Two-pences.

Another Question of the same Nature with the last be this following, viz.

Quest. 12. A Merchant is desirous to change 148 l. into Pieces of 13 d. $\frac{1}{2}$, of 12 d. of 9 d. and 6 d. of 4 d. and he will have of each sort an equal Number of Pieces, I desire to know the Number?

Do as you were taught in the last Question, viz. add several Pieces together, and reduce the Sum into Pence, then reduce the Sum to be charged, viz. 148 l. into the same Denomination, and divide the Greater by the Lesser, and in the Quotient you will find the Answer, viz. 798 is the Number of each of the Pieces required, and 18 remaineth, which is 18 Half-pence by the 8th of this Chapter. See the Work as followeth:

$\begin{array}{r} \text{l.} \\ 148 \\ \hline 240 \text{ Pence in a Pound} \\ \hline 5920 \\ 296 \\ \hline \end{array}$

$\begin{array}{r} d. \\ 13 \text{ } \frac{1}{2} \\ 12 \\ 9 \\ 6 \\ 4 \\ \hline \end{array}$

$\begin{array}{r} 35520 \text{ Pence in } 148 \text{ l.} \\ 2 \\ \hline \end{array}$

$\begin{array}{r} \text{Sum } 44 \\ 2 \text{ } \frac{1}{2} \\ \hline \end{array}$

71040 Half-pence

89 Half-pence.

89) 71040 (797 Pieces of each Sort.

623

874
801

730
712

Rem. (18) Half-pence.

The Truth of the two foregoing Operations will thus be proved, viz. Multiply the Answer by the Parts or Pieces into which the given Number was reduced, and having added the several Products together, if their Sum be equal to the given Number, the Answer is right, otherwise not; so the Answer to the 11th Question I did was 87720; which is proved as followeth. viz.

$\begin{array}{r} \text{l.} \\ 87720 \left\{ \begin{array}{l} \text{Six Pences make } \text{---} 2193 \\ \text{Four Pences make } \text{---} 1562 \\ \text{Two Pences make } \text{---} 731 \end{array} \right. \\ \hline \end{array}$

The total Sum of them 4386 which was the Sum given to be changed.

The Answer to the 12th Question was 798, and 18 Half-pence remained after the Work was ended, now the Truth of the Work may be proved as the Former, viz.

E

798

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Pieces of 13 $\frac{1}{2}$ make—	41	17	09
Pieces of 12 make —	39	18	00
798 { Pieces of 9 make	29	18	00
Pieces of 6 make —	19	19	00
Pieces of 4 make	12	05	00
and 18 Half-pence, or 9 <i>d.</i> remain —	00	00	09

The total Sum of them— 148 00 00
 which total Sum is equal to the Number that was first
 given to be changed, and therefore the Operation was
 rightly performed.

Reduction of Troy-weight.

We come now to give the Learner a few Examples in
 Troy-weight; in working whereof he must be mindful
 of the Table of Troy-weight delivered in the second
 Chapter of this Book.

Quest. 13. In 482 *l.* 7 *oz.* 13 *p.w.* 21 *gr.* how many
 Grains?

Multiply by 12, by 20,
 and by 24, taking in the
 Figures standing in the se-
 veral Denominations, ac-
 cording to the Direction
 given in the Seventh Rule
 of this Chapter, and you
 will find the Product to be
 2780013 Grains, which is
 the Number required, or
 Answer to the Question.
 See the whole Work, as in
 the Margent.

<i>l.</i>	<i>oz.</i>	<i>p. w.</i>	<i>gr.</i>
482	7	13	21
12			
971			
482			
5791	Ounces		
20			
115833	Penny-weight		
24			
463333			
231668			

Facit 2780013 Grains

Quest. 14. 2780013 Grains, I demand how many
 Pounds, Ounces, Penny-weights, and Grains?

This is but the foregoing Question inverted, and is
 resolved by dividing by 24, by 20, and by 12, and
 the Answer is 482 *l.* 7 *oz.* 13 *p.w.* 21 *gr.*

$$\begin{array}{r} 2\text{ } 0 \quad 12) \\ 2780013 \quad (11583\text{ } 3 \quad (5791 \quad (482 \text{ } 1. \end{array}$$

$$\begin{array}{r} 24 \quad 10 \quad 48 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \quad 15 \quad 99 \\ 24 \quad 14 \quad 96 \\ \hline \end{array}$$

$$\begin{array}{r} 140 \quad 18 \quad 31 \\ 128 \quad 18 \quad 24 \\ \hline \end{array}$$

$$\begin{array}{r} 200 \quad 3 \text{ Rem. } 7 \text{ Ounces.} \\ 192 \quad 2 \end{array}$$

81 Rem 13 Penny-weight.

$$\begin{array}{r} 72 \\ \hline \end{array}$$

$$\begin{array}{r} 93 \\ \hline \end{array}$$

$$\begin{array}{r} 72 \\ \hline \end{array}$$

$$\begin{array}{r} \text{L. oz. p.w. gr.} \\ \text{Facit } 482 \quad 7 \quad 13 \quad 21 \end{array}$$

Remain 21 Grains.

Quest. 15. A Merchant sent to a Goldsmith 16 Ingots of Silver, each containing in Weight 2 l. 4 oz. and ordered it to be made into Bowls of 2 l. 8 oz. per Bowl, and Tankards of 1 l. 6 oz. per Piece, and Salts of 10 oz. 10 p.w. per Salt, and Spoons of 1 oz. 18 p.w. per Spoon, and of each an equal Number; I desire to know how many of each Sort he must make?

This Question is of the same Nature with the 11th and 12th Questions foregoing, and may be answered after the same Method, viz. First, add the Weight of the several Vessels, into which the Silver is to be made, into one Sum, and reduce to one Denomination, and they make 1248 Penny-weights; then reduce the Weight of the Ingot into the same Denomination, viz. Penny-weights, and it makes 560 Penny weights, and multiply them by the Number of Ingots, viz. 16, and the Product will give you the Weight of the 16 Ingots viz. 8960; then divide the Product by the Weight of the Vessels, viz. 1248, and the Quotient giveth you the Answer to the Question, viz. 7. and 224 p. w. remaineth over and above.

l. oz.		l. oz. p.w.		
2	4	2	08	00
12		1	06	00
<hr/>		0	10	10
28		0	01	18
20		<hr/>		
<hr/>		Sum	5	02 08
560	Penny-weights	12		
16	Ingots	<hr/>		
<hr/>		62		
3560		20		
560		<hr/>		
<hr/>		1248		
1248)	8960 (7 Vessels of each			
	8736			
<hr/>				

Rem. (224 Penny-weight.

The Proof of the Work is as followeth, viz.

		l.	oz.	p.w.		l.	oz.	p.w.
7 {	Bowls of	2	08	00	per Bowl is	18	06	00
	Tank. of	1	06	00	per Tank. is	10	05	00
	Salts of	0	10	16	per Salt, is	06	16	10
	Spoons of	0	01	18	per Spoon, is	01	01	06
	224 Penny-weight remaining					00	11	04
						<hr/>		
						37	04	00
						<hr/>		

So that you see the Sum of the Weight of each Vessel, together with the Remainder, is 37 l. 4 oz. which is equal to the Weight of the 16 Ingots delivered. For if 37 l. 4 oz. be reduc'd to Penny-weights, it makes 8960.

Reduction of Averdupois-weight.

In reducing Averdupois-weight, the Learner must have Recourse to the Table of Averdupois-weight, delivered in the second Chapter.

Quest. 16. In 47 C. 2 qr. 20 l. how many Ounces? Multiply by 4, by 28, and 16, and the last Product will be the Answer. viz. 84992 Ounces. See the Margent.

$$\begin{array}{r}
 \text{C. qr. l.} \\
 47 \quad 1 \quad 20 \\
 \underline{4} \\
 189 \text{ Quarters} \\
 28 \\
 \hline
 1512 \\
 380 \\
 \hline
 5312 \text{ l.} \\
 16 \\
 \hline
 31872 \\
 5312 \\
 \hline
 \end{array}$$

Facit 84292 Ounces.

Quest. 17. In 84992 Ounces, I demand how many C. qrs. l. and oz.

This is the foregoing Question inverted, and will be resolved, if you divide by 16, by 28, and by 4, and the Answer is 47 C. 1 qr. 20 l. equal to the given Number in the foregoing Question.

$$\begin{array}{r}
 \begin{array}{r} 28 \overline{) 84992} \\ 28 \end{array} \quad \begin{array}{r} 4 \overline{) 5312} \\ 4 \end{array} \quad \begin{array}{r} \text{C. qr. l. oz.} \\ 47 \quad 1 \quad 20 \quad 00 \end{array} \\
 \begin{array}{r} 80 \\ \hline 49 \\ 48 \\ \hline 19 \\ 16 \\ \hline 32 \\ 32 \\ \hline (0) \end{array} \quad \begin{array}{r} 28 \\ \hline 251 \\ 224 \\ \hline 272 \\ 252 \\ \hline (20) \text{ Pounds.} \end{array} \quad \begin{array}{r} 16 \\ \hline 29 \\ 28 \\ \hline (1) \text{ qr.} \end{array}
 \end{array}$$

Reduction of Liquid Measure.

Quest. 28. In 45 Tun of Wine, how many Gallons Multiply by 4, and by 63, the Product is 11340 Gallons for the Answer.

$$\begin{array}{r}
 45 \\
 4 \\
 \hline
 180 \\
 63 \\
 \hline
 540 \\
 1080 \\
 \hline
 \hline
 \end{array}$$

Facit 11340 Gallons.

Quest. 19. In 34 Rundlets of Wine, each containing 18 Gallons, I demand how many Hogsheads?

First, find how many Gallons are in the 34 Rundlets, which you may do, if you multiply 34 by 18, the Content of a Rundlet, and the Product is 612 Gallons, which you may reduce into Hogsheads, if you divide them by 63, and the Quote will be 9 Hogsheads, and 45 Gallons. See the Work.

$$\begin{array}{r}
 34 \\
 18 \\
 \hline
 272 \\
 34 \\
 63 \overline{) 612} \quad (9 \text{ hdds}
 \end{array}$$

567 *Facit* 9 hhds, 45 Gallons

Rem. 45 Gallons.

Quest. 20. In 12 Tun, how many Rundlets of 14 Gallons per Rundlet?

Reduce your Tuns into Gallons, and divide them by 14, the Gallons in a Rundlet, and the Quotient, 216 is your Answer. See the Work following.

$$\begin{array}{r}
 12 \\
 4 \\
 \hline
 48 \\
 63 \\
 \hline
 144 \\
 28 \\
 \hline
 \hline
 \end{array}$$

14) 3024 (216 Randless

$$\begin{array}{r}
 \dots \\
 28 \\
 \hline
 22 \\
 14 \\
 \hline
 84 \\
 84 \\
 \hline
 \hline
 \end{array}$$

(c) Facit 216 Rand.

Reduction of Long-measure.

Quest. 21. I demand how many Furlongs, Poles, Inches, and Barly-corns will reach from London to York, it being accounted 151 Miles?

151 Miles

8 Furlongs in a Mile

1208 Furlongs

40 Poles in a Furlong

48320 Poles

11 Half-yards in a Pole

48320

48320

531520 Half-yards

18 Inches in Half a Yard

4252160

531520

9567360 Inches

3 Barly-corns in one Inch

Facit 28702080 Barly-corns in 151 Miles.

E 4

Quest.

Quest. 22. The Circumference of the Earth (Circles are) is divided into 360 Degrees, at every Degree into 60 Minutes, which (upon the Superficies of the Earth) are equal to 60 Miles; now I demand how many Miles, Furlongs, Perches, Yards, Feet, and Barly-corns will reach round the Globe of the Earth?

360 Degrees

60 Minutes or Miles in a Degree

21600 Miles about the Earth

8 Furlongs in a Mile

172800 Furlongs about the Earth

40 Perches in a Furlong

6912000 Poles or Perches about the Earth

11 Half-yards in a Perch

6912000

6912000

2) 76032000 Half Yards upon the Earth

(38016000 Yards, viz. the Half-yards.

3 Divided by 2

114048000 Feet about the Earth

12 Inches in a Foot

228026000

114048000

1361576000 Inches about the Earth.

3 Barly-corns in an Inch

Fa. 4105728000 Barly-corns.

many will reach round the World, the whole being 21600 Miles; so that if any Person were to go round, and go 15 Miles every Day, he would go the whole Circumference in 1440 Days, which is 3 Years, 11 Months, and 15 Days.

Reduction of Time.

Quest 23. In 28 Years, 24 Weeks, 4 Days, 16 Hours, 30 Minutes, how many Minutes?

<i>Years</i>	<i>Weeks</i>	<i>Days</i>	<i>Hours</i>	<i>Minutes</i>
28	24	4	16	30
52 Weeks in a Year				
<hr/>				
60				
<hr/>				
142				
<hr/>				
1480 Weeks				
<hr/>				
7				
<hr/>				
10364 Days				
<hr/>				
24				
<hr/>				
41462				
<hr/>				
20729				
<hr/>				
248752 Hours				
<hr/>				
60				
<hr/>				
14925150 Minutes				

Note, That in resolving the last Question after the Method expressed, there is lost in every Year 30 Hours. For the Year consisteth of 365 Days and 6 Hours; but by multiplying the Years by 52 Weeks, which is 364 Days, you lose 1 Day and 6 Hours every Year; wherefore to find an exact Answer, bring the odd Weeks, Days, and Hours into Hours, and then multiply the Years by the Number of Hours in the Year, viz. 8766, and to the Product add the Hours contained in the odd Time, and you have the exact Time in Hours, which bring into Minutes, as before. See the last Question thus resolved:

			Weeks	D
			24	4
			7	
			<hr/>	
	Days	Hours	172	
28	365	6	24	
3766	24		<hr/>	
			694	
172	1466		345	
172	730		<hr/>	
197			4144	Hours
228	8766	Hours in a Year.		
<hr/>				
249592	Hours			
<hr/>				
60				
<hr/>				

14975520 Minutes in 28 Years, and 4144 Hours.

So you see that according to the Methods first used to resolve this Question, the Hours contained in the given Time are 248752, but according to the last, best, or truest Method, they are 249592, which exceeds the former by 840 Hours.

But for most Occasions it will be sufficient to multiply the given Years by 365, and to the Product add the Days in the odd Time, if there be any, and then there will be only a Loss of six Hours in every Year, which may be supplied by taking a fourth Part of the given Years, and adding it to the contained Days, and you have your Desire.

Quest. 24. In 438657540 Minutes, how many Years?
Facit 834 Years, 4 Days, 19 Hours.

	8766	Years	Days	Hours.
60 1386 754 10	(7310959	(834	4	19

42

70128

18

29815

18

26298

6

35199

6

35064

57

24) 115 (4 Days

54

96

35

30

Rem. (19) Hours

54

54

(0)

Quest. 25. I desire to know how many Hours and Minutes it is since the Birth of our Saviour Jesus Christ, being accounted 1722 Years?

This Question is of the same Nature with the 24th foregoing, and after the same manner is resolved, viz. multiply the given Number of Years by 8766, the Product is 15095052 Hours; and that by 60, and the Product is 905703120 Minutes. See the Work.

41722 Years

8766 Hours in a Year

10332

10332

12054

13776

15095052 Hours in 1722 Years

60

955703120 Minutes in 1722 Years.

Note

Note, That as Multiplication and Division do interchangeably prove each other, so Reduction Descending and Ascending, prove each other by inverting the Question, as the 13th and 14th, and likewise the 16th and 17th Questions foregoing by Inversion, do interchangeably prove each other. The like may be performed for the Proof of any Question in Reduction whatsoever.

C H A P. IX.

Of Comparative Arithmetick; viz. The Relation of Numbers one to another.

1. **C**omparative Arithmetick, is that which is wrought by Numbers, as they are considered to have Relation one to another, and this consists either in Quantity or in Quality. Vide *Boetius's Arith. Lib. 1. cap. 21.*

2. Relation of Numbers in Quantity, is the Reference or Respect that the Numbers themselves have to one another, where the Terms or Numbers propounded are always two, the first called the Antecedent, and the other the Consequent (See *Wing Arithm.*)

3. The Relation of Numbers in Quantity consists in the Differences, or in the Rate or Reason that is found betwixt the Terms propounded, the Differences of two Numbers being the Remainder found by Subtraction, (according to *Alsted*) but the Rate or Reason betwixt two Numbers is the Quotient of the Antecedent divided by the Consequent, so 21 and 7 being given, the Difference betwixt them will be found to be 14, but the Rate or Reason that is betwixt 21 and 7, will be found to be triple Reason, for 21 divided by 7, quotes 3, the Reason or Rate.

4. The Relation of Numbers in Quality (otherwise called Proportion) is the Reference or Respect that the Reason of Numbers have one unto another; therefore the Terms given ought to be more than two. Now the Proportion or Reason between Numbers relating one to another, is either arithmetical or Geometrical.

5. Arithmetical Proportion is, when divers Numbers differ one from another by equal Reason, that is, have equal Differences.

So the Rank of Numbers, 3, 5, 7, 9, 11, 13, 15, 17, differ by equal Reason, viz. by 2, as you may prove.

6. In a Rank of Numbers that differ by Arithmetical Proportion, the Sum of the first and last Term being multiplied by half the Number of Terms, the Product is the total Sum of all the Terms.

Or, if you multiply the Number of the Terms by the half Sum of the first and last Terms, the Product is the total Sum of all the Terms

So in the former Progression given, 3 and 17 is 20, which multiplied by 4, viz. Half the Number of Terms, the Product gives 80, the Sum of all the Terms; or multiply 8 (the Number of Terms) by 10 (half the Sum of the first and last Term, the Product give 80, as before.

So also, 21, 18, 15, 12, 9, 6, 3, being given, the Sum of all the Terms will be found to be 84; for here the Number of Terms is 7, and the Sum of the first and last (viz. 21 and 3) is 24, half whereof (viz. 12) multiplied by 4, produceth 84, the Sum of the Terms sought.

7. Three Numbers that differ by Arithmetical Proportion, the Double of the Mean (or middle Number) is equal to the Sum of the Extreames.

So 9, 12, and 15, being given, the Double of the Mean 12, (viz. 24) is equal to the Sum of the two Extreames 9 and 15.

8. Four Numbers that differ by Arithmetical Proportion (either continued or interrupted) the Sum of the two Means is equal to the Sum of the two Extreames.

So 9, 12, 18, 21, being given, the Sum of 12 and 18, will be equal to the Sum of 9 and 21, viz. 30; also 6, 8, 14, 16, being given, the Sum of 8 and 14 is equal to the Sum of 6 and 16, viz. 22, &c. See Wingate's Arith. c. 35.

9 Geometrical Proportion (by some called Geometrical Progression) is when divers Numbers differ, according to right Reason.

So 1, 2, 4, 8, 16, 32, 64, &c. differ by double Reason; and 3, 9, 27, 81, 243, 729, differ by triple Reason; 4, 16, 64, 256, &c. differ by quadruple Reason, &c.

10. In any Numbers that increase by Geometrical Proportion, if you multiply the last Term by the ~~Quotient~~ any one of the Terms divided by another of the Terms, which being less is next unto it, and having deducted, or subtracted the first Term out of that Product, divide the Remainder by a Number that is an Unit less than the said Quotient, the last Quote will be the Sum of all the Terms.

So 1, 2, 4, 8, 16, 32, 64, being given, first I take one of the Terms, viz. 8, and divide it by the Term which is less, and next to it (viz. by 4) and the Quotient is 2, by which multiply the last Term 64, and the Product is 128, from whence I subtract the first Term, (viz. 1) the Remainder is 127, which divided by the Quotient 2 made less by 1, viz. 1. the Quote is 127, for the Sum of all the given Terms, as by t Work in the Margent.

$$\begin{array}{r}
 64 \\
 4) 8 \quad (2 \\
 \hline
 128 \\
 1 \\
 \hline
 1) 127 \quad (127 \\
 \hline
 \end{array}$$

So if 4, 16, 64, 256, 1024. were given, the Sum of all the Terms will be found to be 1364. For first, I divide 64, one of the Terms, by the next lesser Term, and the Quotient is 4, by which I multiply the last Term 1024, and it produceth 4096; from whence I subtract the first Term 4, and the Remainder is 4092, which I divide by the Quote less by 1, viz. 3 and the Quote is 1364, for the total Sum of all the Terms, as per Margent.

$$\begin{array}{r}
 1024 \\
 16) 64 \quad (4 \\
 \hline
 4096 \\
 4 \\
 \hline
 3) 4092 \quad (1364 \\
 \hline
 \end{array}$$

11. Three Geometrical Proportionals given, the Square of the Mean is equal to the Rectangle, or Product of the Extreams

So 8, 16, 32, being given, the Square of the Mean, viz. 16, is 256, which is equal to the Product of the Extreams 8 and 32, for 8 Times 32 is equal to 256.

12 Of Four Geometrical proportionable Numbers given, the Product of the two Means is equal to the Product of the two Extreams.

So 8, 16, 32, 64, being given, I say, that the Product of the two Means, viz. 16 Times 32, which is 512, is equal to 8 times 64, the Product of the Extreams.

Also

Also if 3, 9, 21, 63, be given, which are interrupted, I say, 9 Times 21 is equal to 3 Times 63, which is equal

From hence ariseth that precious Gem in Arithmetick, which for the Excellency thereof is called the *Golden Rule*, or *Rule of Three*.

CHAP. X.

The Single Rule of Three Direct.

1. **T**HE Rule of Three (not undeservedly called the *Golden Rule*) is that by which we find out a fourth Number in Proportion unto three given Numbers, so as this fourth Number that is sought may bear the same Rate, Reason, and Proportion to the third (given) Number, as the second doth to the first, from whence it is called the *Rule of Proportion*.

2. Four Numbers are said to be proportional, when the first containeth, or is contained by the second, as often as the third containeth, or is contained by the fourth. Vide *Wingate's Arith.* Chap. 8. Sect. 4.

So these Numbers are said to be Proportionals, viz. 3, 6, 9, 18; for as often as the first Number is contained in the second, so often is the third contained in the fourth, viz. twice. Also 9, 3, 15, 5, are said to be Proportionals; for as often as the first Number containeth the second, so often the third Number containeth the fourth, viz. 3 Times.

3. The Rule of Three is either Simple or Compound.

4. The Simple (or Single) Rule of Three, consisteth of 4 Numbers, that is to say, it hath 3 Numbers given to find out a Fourth, and this is either Direct or Inverse. Vide *Alstead Math.* lib. 2. c. 13.

5. The Single Rule of Three Direct, is when the Proportion of the first Term is to the second, as the third is to the fourth, or when it is required that the Number sought, (viz.) the fourth Number must have the same Proportion to the second, as the third hath to the first.

6. In the Rule of Three, the greatest Difficulty is to discover the Order of the 3 Terms of the Question propounded,

pounded, viz. which is the first, second, and the third; which that you may understand, observe, that the three given Numbers, two always are of one Kind, and the other as of the same Kind with the proportional Number that is sought; as in this Question, viz. If 4 Yards of Cloth cost 12 Shillings, what will 6 Yards cost at that Rate? Here the two Numbers of one Kind are 4 and 6, viz. they both signify so many Yards, and 12 s. is the same Kind with the Number sought, for the Price of 6 Yards is sought.

Again observe, That of the 3 given Numbers, those two that are of the same Kind, one of them must be the first, and the other the third, and that which is of the same Kind with the Number sought, must be the second Number in the Rule of Three; and that you may know which of the said Numbers to make your first, and which your third, know this, that to one of these two Numbers, there is always affixed a Demand, and that Number upon which the Demand lieth, must always be reckoned the third Number. As in the forementioned Question, the Demand is affixed to the Number 6, for it is demanded, what 6 Yards will cost, and therefore 6 must be the third Number, and 4 (which is of the same Denomination or Kind with it) must be the first, and consequently the Number 12 must be the second; and then the Numbers being placed in the formentioned Order, will stand as followeth, viz.

yards	s.	yards
4	12	6

7. The next Thing is, to find out the fourth Number in Proportion; which that you may do, multiply the second Number by the third. and divide the Product thereof by the first, or (which is all one) multiply the 3d Term (or Number) by the second, and divide the Product thereof by the first, and the Quotient thence arising is the 4th Number in a direct Proportion, and is the Number sought, or Answer to the Question, and is of the same Denomination that the second Number is of. As thus, Let the same Question be again repeated, viz. If 4 Yards of Cloth cost 12 Shillings, what will 6 Yards cost?

Having placed my Numbers according to the sixth Rule (of this Chapter) foregoing, I multiply (the second Number) 12, by (the third Number) 6, and the Product is 72, which Product I divide by (the first Number 4, and the Quotient thence arising is 18, which is the 4th Proportional or Number sought, viz. 18 Shillings, (because the second Number is Shillings) which is the Price of 6 Yards, as was required by the Question. See the Work following.

	yds	s.	yds	s.
If	4	12	6	18
		6		

4) 72 (18 Shillings

4

32

32

(0)



Quest. 2. Another Question may be this, viz. If 7 C. of Pepper cost 28 l. how much will 16 C. cost at that Rate?

To resolve which Question, I consider that (according to the sixth Rule of this Chapter) the Terms or Numbers ought to be placed thus, viz. the Demand lying upon 16 C. it must be the third Number, and that of the same Kind with it must be the first, viz. 7 C. and 21 l. (being of the same Kind with the Number sought) must be the second Number in this Question; then I proceed according to this seventh Rule, and multiply the second Number by the third, viz. 21 by 16, and the Product is 336, which I divide by the first Number 7, and the Quotient is 48 l. which is the Value of 16 C. of Pepper at the Rate of 21 l. for 7 C. See the Work following.

c.	l.	c.
7	21	16

16

126

21

7) 336 (48 l.

28

56

56 Facit 48 l.

(0)

8. If when you have divided the Product of the 2d and 3d Numbers by the first, any Thing remain after Division is ended, such Remainder may be multiplied by the Parts of the next inferior Denomination, that are equal to an Unit (or Integer) of the second Number in the Question, and the Product thereof divide by the first Number in the Question, and the Quotient is of the same Denomination with the Parts by which you multiplied the Remainder, and is Part of the 4th Number which is sought, And furthermore, if any Thing remain, after this last Division is ended, multiply it by the Parts of the next inferior Denomination equal to an Unit of the last Quotient, and divide the Product by the same Divisor, (*viz.* The first Number in the Question) and the Quote is still of the same Denomination with your Multiplier; follow this Method until you have reduced your Remainder into the lowest Denomination, &c. An Example or two will make this Rule very plain, which may be the following.

Quest. 3. If 13 Yards of Velter, &c. cost 21 l. what will 27 Yards of the same cost at that Rate?

Having ordered and wrought my Numbers according to the 6th and 7th Rules of this Chapter, I find the Quotient to be 43 *l.* and there is a Remainder of 8, so that I conclude the Price of 27 Yards to be more than 43 *l.* and to the Intent that I may know how much more, I work according to the foregoing Rule, viz. I multiply the said Remainder 8 by 20 *s.* (because the second Number in the Question was Pounds) and the Product is 160, which divided by the first Number, viz. 13. it quotes 12. which are 12 Shillings; and there is yet a Remainder of 4, which I multiply by 12 Pence (because the last Quotient was Shillings) and the Product is 48, which I divide by 13 (the first Number) and the Quotient is 3 *d.* and yet there remaineth 9, which I multiply by 4 Farthings, and the Product is 36, which divided by 13 again, it quotes 2 Farthings, and there is yet a Remainder of 10, which (because it cometh not to the Value of a Farthing) may be neglected; or rather set after the 2 Farthings over the Divisor, with a Line between them; and then (by the 21st and 22d Definitions of the first Chapter of this Book) it will be $\frac{10}{13}$ of a Farthing, so that I conclude, that if 13 Yards of Velvet cost 21 *l.* 27 Yards of the same will cost 43 *l.* 12. 3 *d.* 2 $\frac{10}{13}$ *qrs.* which Fraction is 10 Thirteens of a Farthing. See the Operation as followeth;

The Single Rule

Chap 10.

yds. l. yds.
If 13 21 27

27

147

42

13) 567 (43 l.

..

52

47

39

Remain (8)

Multiply 20

13 160 (12

..

13

30

26

Remain (4)

Multiply 12

13) 48 (3 d.

39

Remain (9)

Multiply 4

— qrs

13) 36 (2 $\frac{10}{13}$

26

Remain 10 Facit 43 l. s. d. qrs

12 3 2 $\frac{10}{13}$

Quest. 4. Another Example may be this following, viz
If 14 Pound of Tobacco cost 27 s. what will 478 Pounds
cost at that Rate?

Chap. 10. of Three Direct.

93

Work according to the last Rule, and you will find it to amount to 92 l. 1 s. 10 d. 1 $\frac{1}{4}$ qrs. and by the 5th Rule of the 8th Chapter 92 l. 1 s. 10 d. 1 $\frac{1}{4}$ qrs. be reduced to 45 l. 1 s. So but then the whole Value of the 478 l. will be 45 l. 1 s. 10 d. 1 $\frac{1}{4}$ qrs. The Work followeth.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
If	14	27	478
			27

3346
956

14) 12906 (92 l. 1 s. 10 d. 1 $\frac{1}{4}$ qrs.)

126	8
30	12
28	12
26	(1) s.
14	

Remains (12)
Multiply 12

24
12

14) 144 (10 d. 1 $\frac{1}{4}$ qrs.)

14

Remains (4)
Multiply 4

14) (16) 1 $\frac{1}{4}$ qrs.

14

Remains (2)

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Facit	46	1	10 1 $\frac{1}{4}$

In

9. In the Rule of Three it many times happens, though the first and third Numbers be of the Kind, as both Money, Weight, Measure, &c. yet they may not be of one Denomination, or perhaps they may both consist of many Denominations; in which Case you are to reduce both Numbers to one Denomination; and likewise your second Number (if it consisteth at any Time of divers Denominations) must be reduced to the least Name mentioned, or lower if you please, which being done, multiply the second and third together, and divide by the first, as is directed in the 7th Rule of this Chapter.

And note, that always the Answer to the Question is in the same Denomination that your second Number is of, or is reduced to, as was hinted before.

Quest. 5. If 15 Ounces of Silver be worth 3 l. 15 s. what are 86 Ounces worth at that Rate?

In this Question the Numbers being ordered according to the 6th Rule of this Chapter, the first and third Numbers are Ounces, and the second Number is of divers Denominations, viz. 3 l. 15 s. which must be reduced to Shillings, and the Shillings multiplied by the third Number, and the Product divided by the first, gives you the Answer in Shillings, viz. 430 Shillings, which are reduced to 21 l. 10 s.

oz.	l.	s.	oz.
If 15	3	15	86
20			
<hr/>			
75			
86			
<hr/>			
450			
600			
<hr/>			
15) 6450	(21) l.	s.	
...	(43) 10		
60			
<hr/>			
45			
(0)			

	4
	<hr/>
	3
	*(10) s.

In resolving the last Question the Work would have been the same, if you had reduced your second Number into Pence, for then the Answer would have been 5160 Pence, equal to 21 l. 10 s. or if you had reduced the second Number into Farthings, the Quotient or Answer would have been 20640 Farthings, equal to the same, as you may prove at your Leisure.

Quest. 6. If 8 l. of Pepper cost 4 s. 8 d. what will 7 C. 3 qrs. 14 l. cost?

In this Question the first Number is 8 l. and the third is 7 C. 3 qrs. 14 l. which must be reduced to the same Denomination with the first, viz. into Pounds, and the second Number must be reduced into Pence; then multiply and divide according to the 7th Rule foregoing, and you will find the Answer to be 6174 Pence, which is reduced into 25 l. 14 s. 6 d.

l. s. d. C. qrs. l.
If 8 cost 4 8 what will 7 3 14 cost?

$$\begin{array}{r} 12 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 4 \\ \hline 31 \\ 28 \\ \hline 152 \\ 63 \\ \hline 882 \end{array}$$

so second Number.

	5292					
	4410					
8)	49392	12)	210)	l.	s.	d.
	(6174	(5114	(25	14	6
	48	60	4			
	18	17	11			
	8	12	10			
	59	54	(14)s.			
	56	48				
	32	(6 d.				
	32					
		l. s. d.				
(0) Facit	25	14	6			

Quest.

Quest. 7. If 3 C. 1 qr. 14 l. of Raisins cost 9 l. 9 s. what will 6 C. 3 qrs. 20 l. of the same cost?

Here the first and third Numbers each consist of diverse Denominations, but must be brought both into one Denomination, &c. as you see in the Operation that followeth, the Answer is 388 s. which is reduced into 19 l. 8 s.

C. qr. l.	l. s.	C. qr. l.
If 3 1 14 cost	9 9	what will 6 3 20 cost?
4	20	4
<hr/>	<hr/>	<hr/>
13	189	27
28		28
<hr/>		<hr/>
108		216
27		56
<hr/>		<hr/>
378 Pounds		776 Pounds

189 Second Number

6984	
6208	
776	
<hr/>	20
378) 146664	(38) 8
	2
1134	<hr/>
<hr/>	18
3326	18
3024	<hr/>
<hr/>	(8)
3024	
3204	
<hr/>	
(0)	

l. s.
Facit 19 8

Quest. 8. If in 4 Weeks I spend 13 s. 4 d. how long will 53 l. 6 s. last me at that rate?

Answer. 2238 Days, equal to 6 Years, 48 Days. the Work.

Pen
it is
H
36
No
is the
73 ca
than
ings

s.	d.	w.	l.	s.
If 3	4	require 4	what will 53	6 cost?
12		7	20	
30		28 Days	1066	
13			12	
160			2132	
			1066	

12792 Pence
28 Second Number

102336
25584
— 3 95)
160) 35817/6 (2238 (6 Years
.... 2190
32
— Rem. (48) Days

38
32

61
48

ye.
Facit 6

days
48 $\frac{26}{100}$

137
128

Remains (96)

Quest. 9. Suppose the yearly Rent of a House, a yearly Pension, or Wages, be 73 l. I desire to know how much it is per Day?

Here you are to bring the Year into Days, and say, if 365 Days require 73 l. what will one Day require?

Now when you come to multiply 73 by 1, the Product is the same; for one neither multiplieth nor divideth, and 73 cannot be divided by 365, because the Divisor is bigger than the Dividend; wherefore bring the 73 l. into Shillings, and they make 1460, which divide by the first

F

Num.

Number 365, and the Quote is 4 Shillings for the Answer:
As you see in the Work.

Days	l.	Day
If 365	73	1
	20	

365)	1460	(4s.
	1460	

		Facit 4 s. per Day.
	(0)	

Quest. 10. A Merchant bought 14 Pieces of Broad-Cloth, each Piece containing 28 Yards, for which he gave after the Rate of 13 s. 6 d. $\frac{1}{2}$ per Yard; now I desire to know how much he gave for the 34 Pieces at that Rate?

First find out how many Yards are in the 14 Pieces, which you will do if you multiply the 14 Pieces by 28 (the Number of Yards in a Piece) and it makes 392; then say, If a Yard cost 23 s. 6 d. $\frac{1}{2}$; what will 392 Yards cost? Work as followeth, and the Answer you will find to be 127400 Half-pence, which reduced, make 265 l. 8 s. 4 d. For after you have multiplied your second and third Numbers together, the Product is 127400, which, according to the seventh Rule) should be divided by the first Number; but the first Number is 1, which neither multiplieth nor divideth, and therefore the Quotient or 4th Number is the same with the Product of the Second and Third; which is in Half-pence, because the Second Number was so reduced. See the Work as followeth.

Facit

and
lib,
that
B
into
are
10 d.
Facit

28

14

112

28

392 Yards in the 14 Pieces

yd. cost s. d. yds,
If 1 cost 13 6 $\frac{1}{2}$, what will 392 cost?
12 325 the second Number

32

13

162

2

Half-pennies 325

1960

784

1176

210

24) 127400 (53018 (265 l.

120

4

74

13

72

12

200

10

192

10

(8) Shillings

l. s. d. Rem: (8) Half-pence, or 4 d.

Facit 265 8 4

Quest. 11. A Draper bought 420 Yards of Broad-cloth, and gave for it after the Rate of 14 s. 10 d. $\frac{2}{4}$ per Ell *Englsh*, now I demand how much he paid for the Whole after that Rate?

Bring your Ells into Quarters, and your given Yards into Quarters, the Ell is 5 Quarters, and in 420 Yards, are 1680 Quarters; then say, if 5 Quarters cost 14 s. 10 d. $\frac{1}{4}$ (or 715 Farthings) what will 1680 Quarters cost? Facit, 250 l. 5 s. See the Operation.

Ells			Yards	
	1		420	
	5		4	
	5		1680 qrs.	
qrs.	s.	d.	1680	
If 5	10	10 $\frac{3}{4}$	715	
	12			
	28		8400	
	15		1680	
	178 d.		11760	
	4		960	
	715 qrs.		5) 1201200 (240240 (2501	
			10	192
			20	482
			20	480
			12 rem. (240) qrs or ss.	
			10	
			20	
			20	
			(0)	
	l. s. d.			
Facit	250 5 0			

Quest. 12. A Draper bought of a Merchant 50 Pieces of Kersey, each Piece containing 34 Ells *English*, (the Ell *Flemish* being three Quarters of a Yard) to pay after the Rate of 8 s. 4 d. per Ell *Flemish*, I demand how much the 50 Pieces cost him at that Rate?

First find out how many Ells *Flemish* are in the 50 Pieces, by multiplying 50 by 34, the Product is 1700 which bring into Quarters by 3, it makes 5100 Quarters, then proceed as in the last Question, and the Answer you will find to be 102000 Pence, or 425 l. See the Operation as followeth.

qrs.	s.	d.			
If 5	8	4	5100	50	
	12		100	34	
				d.	
	100 d. 5)		510000	(10200	200
			...		150
			5		
			10		1700 Ells Fl!
			10		3
					5100
			(0)	(2 0)	
			12) 102000	(850 2	(425 l.
			..		
			96	8	
			60	5	
			60	4	
			(0)	10	
				10	
				(0)	

Facit 425 l.

Quest. 13. A Goldsmith bought a Wedge of Gold, which weighed 14 l. 3 oz. 8 p. w. for the Sum of 514 l. 4 s. I demand what it stood him in per Ounce? Answer, 60 s. or 3 l.

l.	oz.	p. w.	l.	s.	oz.
If 14	3	8	514	4	1
12			20	Shillings	20
31			10284		20 p. w.
14			20 p. w.		
			2 0)		
171 oz. 5428)			205680	(6 0	(3 l.
20			..	6	
			205680		
3428 p. w.				(0)	Facit 60 s. or 3 l.

Quest. 14. A Grocer bought 4 Hhds of Sugar, each weighing neat 6 C. 2 qrs. 14 l. which cost him 2 l. 8 s. 6 d. per C. I demand the Value of the 4 Hhds at that Rate?

First I find the Weight of the 4 Hhds, which you may do by reducing the Weight of one of them into Pounds, and multiply them by 4 (the Number of Hhds) and they make 2968 l. Then say, If 1 C. or 112 l. cost 2 l. 8 s. 6 d. what will 2968 l. cost? *Facit*, 64 l. 5 s. 3 d. As by the Operation.

					C.	qrs.	l.
					6	2	14
					4		
					<hr/>		
					26		
					2		
					<hr/>		
					212		
					53		
					<hr/>		
					742 l. in 1 hhd.		
					4 hogsheds.		
					<hr/>		
					2968 l. in 4 hhds.		
					12)	210)	
					(15413)	12815	(64 l.
					
					12	12	
					<hr/>		
					34	8	
					24	8	
					<hr/>		
					102	(5) Shillings.	
					96		
					<hr/>		
					63		
					60		
					<hr/>		
					(3) Pence.		
					<hr/>		
					l.	s.	d.
					(2) Facit	64	5 3

Quest. 15. A Draper bought of a Merchant 8 Packs of Cloth, each containing 4 Parcels, and each Parcel 10 Pieces, and each Piece 26 Yards, and gave after the Rate of 4 l. 16 s. for 6 Yards, now I desire to know how much he gave for the Whole? *Answer* 6656 l.

First find out how many Yards there were in the 8 Packs, and by the following Work you will find there are 8320 Yards; then say, If 6 Yards cost 4 l. 16 s. what will 8320 Yards cost, &c.

8 Packs.

4

32 Parcels

10

320 Pieces

26

1920

640

8320 Yards.

Tds. l. s. Tds.

6 4 16 8320

20 96

96

49920

74880

6) 798720 (13312|0 (6656 l.

6

12

19

13

18

12

18

11

18

10

7

12

6

12

12

(0)

12

Facit 6656 l.

(0)

E 4.

By

By this time the Learner is, as I suppose well exercised in the Practick and Theorick of the *Rule of Three Direct*; but at his Leisure he may look over the following Questions, whose Answers are given, but the Operation purposely omitted as a Touchstone for the Learner, thereby to try his Ability in what hath been deliver'd in the former Rules.

Quest. 16. If 24 l. of Raisins cost 6 s. 6 d. what will 18 Fraills cost, each weighing neat 3 qrs. 18 l. *Ans.* 24 l. 17 s. 3 d.

Quest. 17. If an Ounce of Silver be worth 5 Shillings, what is the Price of 14 Ingots, each Ingot weighing 7 l. 5 oz. 10 p.w. *Answer* 313 l. 5 s.

Quest. 18. If a Piece of Cloth cost 10 l. 16 s. 8 d. I demand how many Ells *Engl.* there are in the same, when the Ell at that Rate is worth 8 s. 4 d. *Answer.* 26 Ells *English*.

Quest. 19. A Factor bought 84 Pieces of Stuffs, which cost him in all 537 l. 12 s. at 5 s. 4 d. per Yard. I demand how many Yards there were in all, and how many Ells *English* were contained in a Piece of the same? *Answer* 2016 Yards in all, and $19\frac{1}{3}$ Ells of *English* per Piece.

Quest. 20. A Draper bought 242 Yards of Broad-cloth, which cost him in all 254 l. 10 s. for 86 Yards, of which he gave after the Rate of 21 s. 4 d. per Yard. I demand how much he gave per Yard for the Remainder? *Answer* 20 s. 9 d. $\frac{64}{152}$ per Yard.

Quest. 21. A Factor bought a certain Quantity of Serge and Shalloon, which together cost him 126 l. 14 s. 10 d. The Quantity of Serge he bought was 48 Yards, at 4 l. 4 d. per Yard; and for every two Yards of Serge he had 3 Yards of Shalloon; I demand how many Yards of Shalloon he had, and how much the Shalloon cost him per Yard? *Answer.* 120 Yards of Shalloon at 1 l. 15 s. 5 d. $\frac{52}{120}$ per yd.

Quest. 22. An Oilman bought three Tuns of Oil, which cost him 151 l. 14 s. and so it chanced that it leaked out 85 Gallons; but he is minded to sell it again, so that he may be no Loser by it; I demand how he must sell it per Gallon? *Answer,* at 4 s. 6 d. $\frac{174}{871}$ d. per Gallon.

Quest. 23. Bought 9 Packs of Cloth, each Pack containing 12 Cloths, which at 8 s. 4 d. Ell *Flemish*, cost 1050 l. I demand how many Yards there were in each Cloth? *Answer* 27 Yards in each Cloth.

Quest.

Quest. 24. A Gentleman hath 536 *l.* per *Ann.* and his Expences are, one Day with another 18 *s.* 10 *d.* 3 *qrs.* I desire to know how much he layeth up at the Year's End?

Answer 191 *l.* 3 *s.* 8 *d.* 1 *qr.*

Quest. 25. A Gentleman expendeth daily one Day with another 27 *s.* 10 *d.* $\frac{1}{2}$. and at the Year's End layeth up 340 *l.* I demanded how much is his yearly Income? *Answer* 848 *l.* 14 *s.* 4 *d.* $\frac{1}{2}$.

Quest. 26. If I sell 24 Yards for 10 *l.* 10 *s.* how many Ells *Flemish* shall I sell for 283 *l.* 17 *s.* 6 *d.* at that Rate?

Answer 504 $\frac{2}{3}$ Ells *Flemish*.

Quest. 27. If 100 *l.* in 12 Months, gain 6 *l.* Interest, how much will 75 *l.* gain in the same Time, and at the same Rate? *Answer* 4 *l.* 10 *s.*

Quest. 28. If 100 *l.* in 12 Months gain 6 *l.* Interest, how much will it gain in 7 Months at that Rate? *Answer* 3 *l.* 10 *s.*

Quest. 29. A certain Usurer put out 73 *l.* for 12 Months, and received Principal and Interest 81 *l.* I demand what Rate per Cent. he received Interest? *Answer* 8 *l.* per Cent.

Quest. 30. A Grocer bought 2 Chests of Sugar, the one weigh'd near 18 C 3 *qrs.* 14 *l.* at 2 *l.* 6 *s.* 8 *d.* per C. the other weighed near 18 C. 1 *qr.* 21 *l.* at 4 *d.* $\frac{1}{2}$ per *l.* which he mingled together; now I desire to know how much a Cwt. of this Mixture is worth? *Ans.* 2 *l.* 4 *s.* $2\frac{5567}{5567}$ *qrs.*

Quest. 31. Two Men, viz. A and B. departed both from one Place, the one goes East, and the other West the one travelleth 4 Miles a Day, and the other 5 Miles a Day, how far are they distant the 9th Day after their Departure? *Answer* 81 Miles.

Quest. 32. A flying every Day 40 Miles, is pursued the fourth Day after by B, posting 50 Miles a Day; now the Question is, in how many Days, and after how many Miles Travel, will A be overtaken?

Ans. B overtakes him in 32 Days, when they have travelled 600 Miles. See *More's Arithm.* cap. 8. qu 7.

11. The general Effect of the Rule of Three Direct, is contained in the Definition of the same, that is, to find a fourth Number in Proportion, consisting of two equal Reasons; as hath been fully shewn in all the foregoing Examples.

The second Effect is, by the Price or Value of one Thing, to find the Price and Value of many Things of like Kind.

The third Effect is, by the Price or Value of many Things, to find the Price of one; or by the Price of many Things, (the said Price being one) to find the Price of many Things of like Kind.

The 4th Effect is, by the Price or Value of many Things, to find the Price or Value of many Things of like Kind.

The 5th Effect is, thereby to reduce any Number of Monies, Weights, or Measures, the one Sort into the other, as in the Rules of Reduction contained in the 8th Chapter foregoing. Examples of its various Effects have been already answered.

12. The Rule of Three Direct, is thus proved, *viz.* Multiply the first Number by the 4th, [*The Proof of the Rule of Three Direct.*] and note the Product; then multiply the 2d Number by the 3d, and if this Product is equal to the Product of the 1st and 4th, then the Work is rightly performed, otherwise it is erroneous.

So the first Question of this Chapter (whose Answer or 4th Number we found to be 18 s.) is thus proved, *viz.* the first Number is 4, which multiplied by 18 (the 4th) produced 72, and the 2d and 3d Numbers are 12 and 6, which multiplied together produceth 72, equal to the Product of the 1st and 4th, and therefore I conclude the Work to be rightly performed.

Always observing, that if any Thing remain after you have divided the Product of the 2d and 3d Numbers by the first, such Remainder in proving the same, must be added to the Product of the 1st and 4th Numbers, whose Sum will be equal to the Product of the second and third, the second Number being of the same Denomination with the fourth, and the first of the same Denomination with the third.

So the fourth Question of this Chapter being again repeated, *viz.* If 142 l. of Tobacco cost 27 s. what will 478 l. cost at that Rate? The Answer, (or fourth Number) was 46 l. 1 s. 10 d. 1 gr. $\frac{1}{4}$, which is thus proved; *viz.* bring the 4th Number into Farthings, and it makes 44294, which multiplied by the first Number 14, produceth

Chap. 11 *The Single Rule. &c.*

duceth 619488, (the second which remaineth being added thereto;) then (because I reduce my fourth Number into Farthings) I reduce my second, (*viz.* 27 s.) into Farthings, and they are 1296, which multiplied by the 3d Number 478, their Product is 619488, equal to the Product of the first and fourth Numbers: Wherefore I conclude the Operation to be true. This is an infallible Way to prove the Rule of Three Direct, and it is reduced from the 12th Section of the 9th Chapter of this Book.

And thus much for this inestimable Rule of Three Direct, the Demonstration of which may be seen in *Kersey's Appendix to Wingate's Arithm.* and in the 6th Chapter of *Oughtred's Clavis Mathematica*.

C H A P. XI.

The Single Rule of Three Inverse.

1. **T**H E Golden Rule, or Rule of Three, Inverse, is when there are 3 Numbers, given to find a 4th in such Proportion to the 3 given Numbers, so as the 4th proceeds from the 2d according to the same Rate, Reason, or Proportion, that the first proceeds from the third, or the Proportion is,

As the 5th Number is in Proportion to the 2d, so is the 1st to the 4th. See *Alsted. Matth.* l. 2. c. 14.

So if the 3 Numbers given were 8, 12, and 16, and it were required to find a fourth Number in an inverted Proportion to these, I say, that as 16 (the third Number) is the Double of the first Term or Number (8) so mult 12, the second Number, be the double of the fourth; so will you find the fourth Term or Number to be 6. And as in the Rule of Three Direct) you multiply the second and third together, and divide their Product for a fourth proportional Number,

2. In the Rule of Three Inverse, you must multiply the second Term by the first, or first Term, by the second, and divide the Product thereof by the first Term, so the Quotient will give you the 4th Term sought in an inverted Proportion. The same Order being observed in this Rule, as in the Rule of Three Direct, for placing and disposing of the given

given Numbers, and after your Numbers are placed in order, that you may know whether your Question be to be resolved by the Rule Direct or Inverse, observe the general Rule following.

3. When your Question is stated, and your Numbers orderly disposed, Consider in the first Place, whether the fourth Term or Number sought, ought to be more or less than the second Term; which you may easily do: And if it is required to be more or greater than the 2d Term, then the lesser Extream must be your Divisor; but if it requires less, then the highest Extream must be your Divisor; in this Case; the 1st and 3d Numbers are called Extreams (in respect of the second) and having found out your Divisor, you may know whether your Question belong to the Rule Direct or Inverse; for if the 3d Term be your Divisor, then it is Inverse; but if the 1st Term be your Divisor, then it is a direct Rule. As in the following Questions.

Quest. 1. If 8 Labourers can do a certain Piece of Work in 12 Days, in how many Days will 16 Labourers do the same? *Answer*, in 6 Days.

Having placed the Numbers according to the 6th Rule of the 10th Chapter, I consider, that if 8 Men can finish the Work in 12 Days, 16 Men will do it in lesser (or fewer Days) than 12 therefore the biggest Extream must be the Divisor, which is 16, and therefore it is the Rule of Three Inverse; wherefore I multiply the 1st and 2d Numbers together, viz. 8 by 12, and their Product is 96, which divided by 16, quotes 6 Days for the Answer; and in so many Days will 16 Labourers perform a Piece of Work, when 8 Men can do it in 12 Days.

lab.	Days	lab.
8	12	16
	8	
<hr/>		
16)	96	(5 Days
	96	
	(0)	
<i>Facit</i> 6 Days		

Quest. 2. If, when the Measure, viz. (a Peck) of Wheat cost 2 s. the Penny-loaf weighed (according to the Standard Statute or Law of England) 8 Ounces, I demand how much it will weigh when the Peck is worth 1 s. 6 d. according to the same Rate or Proportion? *Answer* 10 oz. 23 p. w. 8 gr.

Having

Having placed and reduced the given Numbers according to the 6th and 9th Rules of the 1th Chapter, I consider that at 1 s. 6 d per Peck, the Penny-loaf will weigh more than at 2 s. per Peck; for as the Price decreaseth, the Weight increaseth; and as the Price increaseth, so the Weight diminishes; wherefore because the first Term requires more than the second, the lesser Extream must be the Divisor. viz. 1 s. 6 d or 18 d. and having finished the Work, I find the Answer to be 10 oz. 13 p. w. 8 gr. and so much will the Penny-loaf weigh when the Peck of Wheat is worth 1 s. 6 d. according to the given Rate of 8 Ounces, when the Peck is worth two Shillings. The Work is plain in the following Operation.

d.	oz.	gr.
If 24	8	18
8		

18) 192 (10 13 8 Answer

18

12

20

240 p. w.

(13

18

60

54

(6)

24

144 (8 Gr.

144

(0)

Quest. 3. How many Pieces of Money or Merchandize at 20 s. per Piece, are to be given or received for 240 Pieces, the Value or Price of every Piece being 12 Shillings? Answer 144 Pieces. For it 12 s. required 240 Pieces, then 20 s. will require less; therefore the bigger Extream must be the Divisor, which is the third Number, &c. See the Work as in the Margent.

s.	per.	s.
If 12	240	20
	12	
	<hr style="width: 50%; margin: 0 auto;"/>	
	480	
	240	
	<hr style="width: 50%; margin: 0 auto;"/>	
210)	28810	(144 pes. at
	... 20 s. per pe.	
	2	
	<hr style="width: 50%; margin: 0 auto;"/>	
	8	
	8	
	<hr style="width: 50%; margin: 0 auto;"/>	
	8	
	8	
	<hr style="width: 50%; margin: 0 auto;"/>	
	(0)	

Quest. 4. How many Yards of 3 Quarters broad are required to double, or be equal in Measure to 30 Yards, that are 5 Quarters broad? Answer 50 Yards. For say, if 5 Quarters will require 30 Yards long, what Length will 3 Quarters broad require? Here I consider that 3 Quarters broad will require more Yards than 30; for the narrower the Cloth is, the more in length will go to make equal Measure with a broader Piece.

qrs.	long	qrs.
3	30	5
	5	
	<hr style="width: 50%; margin: 0 auto;"/>	
3)	250	(50 yds.)
	15	
	<hr style="width: 50%; margin: 0 auto;"/>	
	(0)	

Quest. 5. At the Request of a Friend, I lent him 200 l. for 12 Months; promising to do me the like Courtesy at my Necessity; but when I came to request it of him, he could let me have but 150 l. now I desire to know how long I may keep this Money to make plenary Satisfaction for my former Kindness to my Friend? Answer 16 Months, I say, If 200 l. will require 12 Months, what will 150 l. require; 150 l. will require more Time than 12 Months, therefore the lesser Extream, (viz. 150) must be the Divisor; multiply and divide, and you will find the fourth inverted proportional to be 16, and so many Months I ought to keep the 150 l. for Satisfaction.

Quest. 6. If for 24 s. I have 1200 l. Weight carried 36 Miles, how many Miles shall 1800 l. be carried for the same Money? Answer 24 Miles.

Quest. 7. If for 24s. I have 1200 l. wt. carried 36 Miles, how many l. wt. shall I have carried 24 Miles for the same Money? *Answer* 1800 l. weight.

Quest. 8. If 100 Workmen in 12 Days finish a Piece of Work or Service, how many Workmen are sufficient to do the same in 3 Days? *Answer* 400 Workmen.

Quest. 9. A Colonel is besieged in a Town in which are 1000 Soldiers, with Provision of Victuals only for three Months, the Question is, How many of his Soldiers must he dismiss, that his Victuals may last the remaining Soldiers 6 Months? *Answer* 500 he must keep, and dismiss as many.

Quest. 10. If 20 l. worth of Wine is sufficient for the Ordinary of 100 Men, when the Tun is sold for 30 l. how many Men will the same 20 l. worth suffice when the Tun is worth 24 l. *Answer* 125 Men.

Quest. 11. How much Plush is sufficient for the Cloak, which hath in it 4 Yards of 7 Quarters wide, when the Plush is but 3 Quarters wide? *Answer* $9\frac{1}{2}$ Yds of Plush.

Quest. 12. How many Yards of Canvas that is Ell wide, will be sufficient to line 20 Yards of Say, that is 3 Quarters wide? *Answer* 12 Yards.

Quest. 13. How many Yards of Matting that is 2 Foot wide, will cover a Floor that is 24 Foot long, and 26 Foot broad? *Answer* 240 Foot.

Quest. 14. A Regiment of Soldiers, consisting of 1000, are to have new Coats, and each Coat to contain two Yards two Quarters of Cloth that is 5 Quarters wide, and they are to be lined with Shalloon that is 3 Quarters wide, I demand how many Yards of Shalloon will line them? *Answer* $16666\frac{2}{3}$ Quarters, or $4166\frac{1}{2}$ Yards.

Quest. 15. A Messenger makes a Journey in 24 Days, when the Day is 12 Hours long: I desire to know in how many Days he will go the same when the Day is 16 Hours long? *Answer*, in 18 Days.

Quest. 16. I borrowed of my Friend 64 l. for 8 Months, and he hath Occasion another Time to borrow of me for 12 Months. I desire to know how much I must lend to make good his former Kindness to me? *Answer*, 42 l.

4. The general Effect of the Rule of Three Inverse, is contained in the Definition of the same, that is, to find a fourth Term in a Reciprocal Proportion inverted to the Proportion given.

The 2d Effect is, by two Pieces, or Value of two several Pieces of Money and Merchandizes known, to find how many Pieces of the one Price is to be given for so many of the other. And so to reduce and exchange one Sort of Money or Merchandize into another. Or else to find the Price unknown of any Piece given to exchange in reciprocal Proportion.

The 3d Effect is, by two different Prices of a Measure of Wheat bought or sold, and the Weight of the Loaf of Bread, made answerable to one of the Prices of the Measure given, to find out the Weight of the same Loaf answerable to the other Price of the said Measure given.

Or else, by the two several Weights of the same priced Loaf, and the Price of the Measure of Wheat answerable to one of those Weights given, to find out the other Price of the Measure answerable to the other Weight of the same Loaf.

The 4th Effect is, by two Lengths, and one Breadth of two Rectangular Planes known, to find out another Breadth unknown. Or, by 2 Breadths, and one Length given, to find out another Length unknown in an inverted Proportion.

The 5th Effect is, by double Time, and a capital Sum of Money borrowed or lent, to find out another capital Sum answerable to one of the given Times; or otherwise, by two capital Sums, and a Time answerable to one of them given, to find out a Time answerable to the other capital Sum in reciprocal Reason.

The 6th Effect is, by two different Weights of Carriage, and the Distance of the Place in Miles or Leagues given, to find another Distance in Miles answerable to the same Price of Payment. Or otherwise by two Distances in Miles, and the Weight answerable to one of the Distances (being carried for a certain Price) to find out the Weight answerable to the other Distance for the same Price.

The 7th Effect is, by double Workmen, and the Time
answer

answerable to one of the Numbers of Workmen given, to find out the Time answerable to the other Number of Workmen, in the Performance of any Work or Service. Or contrariwise, by double Time, and the Workmen answerable to one of those Times given, to find out the Number of Workmen answerable to the other Time, in the Performance of any Work or Service.

Also by a double Price of Provision, and the Number of Men, or other Creatures, nourished for a certain Time answerable to one of the Prices of Provisions given, to find out another Number of Men or other Creatures answerable to the other Price of the Provision for the same Time. Or contrariwise, by two Numbers of Men, or other Creatures nourished, and one Price of Provision answerable to one of the Numbers of Creatures given, to find out the other Price of the same Provision answerable to the other Number of Creatures, both being supposed to be nourished for the same, &c.

To prove the Operation of the Rule of Three Inverse, multiply the 3d and 4th Terms together, and note their Product; and multiply the 1st and 2d together, and if their Product is equal to the Product of the 3d and 4th, then is the Work truly wrought, but if it falleth out otherwise, then it is erroneous.

As in the first Question of this Chapter 16 (the 3d Number) being multiplied by 6 (the 4th Number) the Product is 96, and the Product of 8, (the first Number) multiplied by 12, (the 2d Number) is 96, equal to the first Product, which proves the Work to be right.

And note, That if in Division any Thing remain, such Remainder must be added to the Product of the third and fourth Terms, and if the Sum be equal to the Product of the first and second (the Homogeneal Terms being of one Denomination). the Work is right.

C H A P. XII.

The Double Rule of Three Direct.

WE have already delivered the Rule of single Proportion, and we come now to lay down the Rules of Plural Proportion.

1. Plural

1. Plural Proportion is, when more Operations in the Rule of Three than one are required before a Solution can be given to the Question propounded. Therefore in Questions that require Plurality in Proportion, there are always given more than three Numbers.

2. When there are given five Numbers, and a sixth is required in Proportion thereunto, then the sixth Proportion is said to be found out by the Double Rule of Three, as in the Question following, *viz.*

If 100 *l.* in 12 Months gain 6 *l.* Interest, how much will 75 *l.* gain in 9 Months?

3. Questions in the Double Rule of Three, may be resolved either by 2 Single Rules of Three, or by 1 Single Rule of Three, compounded of the Five given Numbers.

4. The Double Rule of Three, is either Direct, or else Inverse.

5. The Double Rule of Three Direct is, when unto 5 given Numbers, a 6th proportional may be found out by two single Rules of Three Direct.

6. The five given Numbers in the Double Rule of Three Direct consist of two Parts, *viz.* 1. A Supposition, and 2dly, of a Demand; the Supposition is contained in the three first of the five given Numbers, and the Demand lies in the two last; as in the Example of the Second Rule of this Chapter, *viz.* If 100 *l.* in 12 Months gain 6 *l.* Interest, what will 75 *l.* gain in 9 Months? Here the Supposition is expressed in 100, 12, and 6, for it is said, if 100 *l.* in 12 Months gain 6 *l.* Interest: And the Demand lieth in 75 and 9, for it is demanded, How much 75 *l.* will gain in 9 Months;

7. When your Question is stated, the next Thing will be to dispose of the given Numbers in due Order and Place, as a Preparative for Resolution: Which that you may do; First, Observe which of the given Numbers in the Supposition is of the same Denomination with the Number required: for that must be the 2d Number (in the first Operation) of the Single Rule of Three, and one of the other Numbers in the Supposition (it matters not which) must be the first Number, and that Number in the Demand, which is of the same Denomination with the first, must be the third Number; which three Numbers being thus placed,

placed, will make one perfect Question in the Single Rule of Three, as in the forementioned Example: First, I consider, that the Number required in the Question, is the Interest or Gain of 75 l. therefore that Number in the Supposition which hath the same Name, (*viz.* 100 6 75. 6 l. which is the Interest or Gain of 100 l.

must be the second Number in the first Operation, and either 100 or 12, (it matters not which) must be the first Number; but I will take a 100; and then for the third Number, I put that Number in the Demand, which hath the same Denomination with 100, which is 75; for they both signify Pounds principal) and then the Numbers will stand as you see in the Margent.

But if I had for the first Number put the other Number in the Supposition, *viz.* 12, which signifies 12 Months, then the third Number must have been 9, which is the Number in the Demand which hath the same Denomination with the first, *viz.* 9 Months, and then they will stand as in the Margent.

There yet remain two Numbers to be disposed of, and those are one in the Supposition, and another in the Demand; that which is of the Supposition, I place under the first of the three Numbers; and the other, which is of the Demand, I place under the third Number; and then two of the Terms in the Supposition will stand (one over the other) in the first Place, and the two Terms in the Demand will stand (one over the other) in the third Place, as in the Margent.

8. Having disposed or ordered the given Numbers, according to the last Rule, we may proceed to a Resolution; and first I work with the 3 uppermost Numbers, which, according to the first Disposition, are 100, 6, and 75; which is as much as to say, If 100 l. requires 6 l. Interest, how much will 75 Pound require? which by the 3d Rule of the 11th Chap. I find to be Direct; and by the 7th and 8th Rules of the 10th Chapter, I find the 4th proportional Number to be 4 l. 10 s. so that by the foregoing single Question I have discovered how much Interest 75 l. will gain in 12 Months; the Operation whereof followeth on the left Hand under the Letter A, and having

disco-

So that by the foregoing Operation, I conclude, that if 100 *l.* in 12 Months gain 6 *l.* Interest, 75 *l.* will gain 3 *l.* 7 *s.* 6 *d.* in 9 Months, after the same Rate.

The Answer would have been the same if the 12 9 9
5 given Numbers had been ordered accord- 100 75
ing to the second Method, viz. as you see
in the Margent.

For first, I say, if twelve Months gain 6 *l.* what will 9 Months gain? This Question I find to be Direct, by the 3d Rule of the 11th Chapter, and by the 7th and 8th Rules of the 10th Chapter, I find the fourth proportional Number to these three to be 4 *l.* 10 *s.*

Thus have I found out what is the Interest of 100 *l.* for 6 Months, and I am now to find the Interest of 75 *l.* for 9 Months; to effect which, I make this 4th Number (found as before) to be my second Number in the next Question, I say, If 100 *l.* require 4 *l.* 10 *s.* what will 75 *l.* require? This Question, I find (by the said 3d Rule of the 11th Chapter) to be Direct, and by the said 7th, 8th, and 9th Rules of the 10th Chapter, I find the Answer to be as before, viz. 3 *l.* 7 *s.* 6 *d.*

The Operation of this Rule in the following Questions, are purposely omitted, to try the Learner's Capacity.

Quest. 2. A 2d Example in this Rule may be as followeth, viz. A Carrier receiving 42 Shillings for the Carriage of 300 Weight 150 Miles, I demand how much he ought to receive for the Carriage of 7 C. 3 *qrs.* 4 *l.* 50 Miles at that Rate? *Answer* 36 *s.* 9 *d.*

Quest. 3. A Regiment of 136 Soldiers eat up 351 Quarters of Wheat in 108 Days, I demand how many Quarters of Wheat 1132 Soldiers will eat in 56 Days at that Rate? *Answer* 1404 Quarters.

Quest. 4. If 40 Acres of Grass be mowed by 8 Men in 7 Days, how many Acres shall be mowed by 24 Men in 28 Days? *Answer* 480 Acres.

Quest. 5. If 48 Bushels of Corn (or other Seed) yield 576 Bushels in a Year, how much will 240 Bushels yield in 6 Years at that Rate? That is to say, if they were sowed 240 Bushels every one of the 6 Years? *Answer* 17280 Bushels.

Quest. 6. If 40 Shillings is the Wages of 8 Men for 5 Days, what will be the Wages of 32 Men for 24 Days?

Answer 768 Shillings, or 38 l. 8 s.

Quest. 7. If 14 Horses eat 46 Bushels of Provender in 16 Days, how many Bushels will 20 Horses eat in 24 Days? *Answer* 120 Bushels.

Quest. 8. If 8 Cannons in one Day spend 48 Barrels of Powder, I demand how many Barrels 24 Cannons will spend in 22 Days at that Rate? *Answer* 1728 Barrels.

Quest. 9. If in a Family consisting of 7 Persons, there are drunk out 2 Kilderkins of Beer in 12 Days, how many Kilderkins will there be drunk out in 8 Days by another Family consisting of 14 Persons? *Answer* 48 Gallons, or 2 Kilderkins and 12 Gallons.

Quest. 10. An Usurer put 75 l. out, to receive Interest for the same, and when it had continued 9 Months, he received for Principal and Interest 78 l. 7 s. 6 d. I demand at what Rate per Cent. per Annum, he received Interest? *Answer* 6 l. per Cent. per Annum.

C H A P. XIII.

The Double Rule of Three Inverse.

THE Double Rule of Three Inverse, is, when a Question in the Double Rule of Three is resolved by two Single Rules of Three, and one of those single Rules falls out to be Inverse, or requires a fourth Number in Proportion reciprocal (for both Questions are never Inverse).

2. In all Questions of the Double Rule of Three (as well Inverse as Direct) you are in the disposing of the 5 given Numbers, to observe the 7th Rule of the 12th Chapter, and in resolving of it by two single Rules, observe to make Choice of your Numbers for the first and single Questions, according to the Directions given in the 8th Rule of the same Chapter, and in the Example following *viz.*

Quest. 1. If 100 l. Principal in 12 Months gain 6 l. Interest, what Principal will gain 3 l. 7 s. 6 d. in 9 Months?

This

This Question is an Inversion of the first Question of the 12th Chapter, and may serve for a Proof thereof.

In order to a Resolution, I dispose of the 5 given Numbers, according to the 9th Rule of the last Chapter; and being so disposed, they will stand as followeth.

12	100	9	<i>l.</i>	<i>s.</i>	<i>d.</i>
6			3	7	6

Or thus,

6	100	<i>l.</i>	<i>s.</i>	<i>d.</i>
12		3	7	6

First I say,

<i>m</i>	<i>l</i>	<i>m</i>
12	100	9
	12	

	<i>l.</i>	<i>s.</i>	<i>d.</i>
9) 1200	(133	6	8
...			

9	<i>l.</i>	<i>s.</i>	<i>d.</i>
—	<i>Fac.</i>	133	6 8

30

27

—

30

27

—

(3)

20

9) 60 (6 *s.*

54

—

(6)

12

—

9) 72 (8 *d.*

72

—

(0)

Here observe, That according to the 8th Rule of the 12th Chapter, the first Question (if you take it from the 5 Numbers (as they are ordered or placed first) will be, if 12 Months require 100 *l.* Principal, what will 9 Months require to make the same Interest? This (according to the 3d Rule of the 12th Chapter) is Inverse, and the Answer will be found (by the 2d Rule of the 11th Chapter) to be 133 *l.* 6 *d.* 8 *d.* The 2d Question then will be, If 6 *l.* Interest require 133 *l.* 6 *s.* 8 *d.* Principal; how much Principal will 3 *l.* 7 *s.* 6 *d.* require? This is a direct Rule, and the Answer in a direct Proportion, 75 *l.* See the Work.

			Then I say,					
	l.	l.	s.	d.	l.	s.	d.	
If	6	133	6	8	3	7	6	
	240	20			20			
	<hr/>				<hr/>			
	1440 d	2566			67			
		12			12			
		<hr/>			<hr/>			
		5340			140			
		2566			67			
		<hr/>			<hr/>			
		32000			8 10 d.			
		810						
		<hr/>						
		32000						
		256						
		<hr/>			2410			
	14410	25920000			(180000 d. or 75 l.			
				
		144			168			
		<hr/>			<hr/>			
		1152			120			
		1152			120			
		<hr/>			<hr/>			
		(0)			(0)			

So that by the foregoing Work I find, that if 6 l. Interest be gained by 100 l. in 12 Months, 3 l. 7 s. 6 d. will be gained by 75 l. in 9 Months.

But if the Resolution had been found out by the Number as they are ranked in the second Place, then the second Question in the Single Rule would have been Inverse, and the first Question Direct, and the Conclusion the same with the first Method, viz. 75 l.

Quest. 2. If a Regiment consisting of 939 Soldiers, can eat up 351 Quarters of Wheat in 168 Days, how many Soldiers will eat up 1404 Quarters in 56 Days at the same Rate? *Answer* 11232 Soldiers.

Quest. 3. If 12 Students in 8 Weeks spend 48 l. I demand how many Students will spend 288 l. in 18 Weeks? *Answer* 32 Students.

Quest. 4. If 48 l. serve 12 Students 8 Weeks, how many Weeks will 288 l. serve 4 Students? *Answer* 144 Weeks.

Quest. 5. If when a Bushel of Wheat cost 3 s. 4 d. the Penny Loaf weigheth 12 Ounces, I demand the Weight of the Loaf worth 9 d. when the Bushel cost 10 s. *Answer* 35 Ounces

Quest. 6. If 48 Pioneers in 12 Days cast a Trench 24 Yards long, how many Pioneers will cast a Trench 168 Yards long in 16 Days? *Answer* 252 Pioneers.

Quest. 7. If 12 C. weight being carried 100 Miles, cost 5 l. 11 s. I desire to know how many C. weight may be carried 150 Miles for 12 l. 12 s. at that Rate? *Ans.* 18 C.

Quest. 8. If when Wine is worth 30 l. per Ton, 20 l. worth is sufficient for the Ordinary of 100 Men, how many Men will 4 l. worth suffice, when it is worth 24 l. per Ton? *Answer* 25 Men.

Quest. 9. If 6 Men in 24 Days mow 72 Acres; in how many Days will 8 Men mow 24 Acres? *Ans.* In 6 Days.

Quest. 10. If when the Ton of Wine is worth 30 l. 100 Men will be satisfied with 20 l. worth, I desire to know what the Ton is worth when 4 l. worth will satisfy 25 Men at the same Rate? *Answer* 24 l. per Ton.

CHAP. XIV.

The Rule of Three composed of Five Numbers.

THE Rule of Three composed, is when Questions (wherein there are 5 Numbers given to find a 6th in Proportion thereunto) are resolved by one single Rule of Three composed of the 5 given Numbers.

2. When Questions may be performed by the Double Rule of Three Direct, and it is required to resolve them by the Rule of Three composed; first order or rank your Numbers according to the 7th Rule of the 12th Chap. then

The Rule is,

Multiply the Terms or Numbers (that stand one over the other in the first Place) the one by the other, and make their Product the first Term in the Rule of Three Direct; then multiply the Terms that stand one over the other in the third Place, and place their Product for the

3d Term, in the Rule of Three Direct, and put the middle Term of the 7 uppermost for a second Term; then having found a 4th Proportional Direct to these three, this 4th Proportional so found shall be the Answer required.

So the first Question of the 12th Chapter being proposed, viz. If 100 *l.* in 12 Months gain 6 *l.* Interest, what will 75 *l.* gain in 9 Months? The Numbers being ranked (or placed) as is there directed and done.

Then I multiply the two first Terms, 100 and 12, the one by the other, and their Product is 1200 for the first Term; then I multiply the two last Terms 75 and 9 together, and their Product is 675 for the third Term. Then I say, as 1200 is to 6, so is 675 to the Answer, which by the Rule of Three Direct, will be found to be 3 *l.* 7 *s.* 6 *d.* as was before found.

3. But if the Question be to be answered by the Double Rule of Three Inverse, then (having placed the 5 given Terms as before) multiply the lowermost Term of the first Place, by the uppermost Term of the third Place, and put the Product for the first Term; then multiply the uppermost Term of the first Place, by the lowermost Term of the third Place, and put the Product for the third Term, and the second Term of the three highest Numbers for the middle Term to those two; then if the Inverse Proportion is found in the uppermost three Numbers, the fourth Proportional Direct to these three shall be the Answer. So the first Question to the 13th Chapter being stated, viz. If 100 *l.* Principal in 12 Months gain 6 *l.* Interest, what Principal will gain 3 *l.* 7 *s.* 6 *d.* in 9 Months? State the Number as there directed in the first Order, viz.

m	l.	m.
12	100	9
l.		l. s. d.
6		3 7 5

Then reduce the 6 *l.* and 3 *l.* 7 *s.* 6 *d.* into Pence, the 6 *l.* is 1440 *d.* and 3 *l.* 7 *s.* 6 *d.* is 810 *d.* then multiply 1440 by 9, the Product is 12960 for the first Term in the Rule of Three Direct, and multiply 810 by 12, the Product is 9720, for the third Term; then I say, as 12960 is to 130, so is 9720 to the Answer, viz. 75 *l.* as before. But if the Terms had been placed after the second Order, viz.

<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
6	100	3	7	6
<i>m</i>		<i>m</i>		
12		9		

Then the Inverse Proportion is found in the lowest Numbers, and having composed the Numbers for a single Rule of Three, as in the second Rule foregoing; then the Answer must be found by a single Rule of Three Inverse; for here it falls out to multiply 810 by 12 for the first Number, 1440 by 9 for the Third Number; and then you must say, As 9720 is to 100 *l.* so is 12960 to the Answer, which by Inverse Proportion will be found to be 75 *l.* as before.

The Question in the 12th and 13th Chapters may serve for thy farther Experience.

C H A P. XV.

Single Fellowship.

Fellowship, is that Rule of Plural Proportion, whereby we ballance Accompts depending between divers Persons having put together a general Stock, so that they may every Man have his proportional Part of Gain, or sustain his proportional Part of Loss.

2. The Rule of Fellowship is either single, or it is Double.

3. The Single Rule is, when the Stocks propounded are single Numbers, without any Respect or Relation to Time, each Partner continuing his Money in Stock for the same Time.

4. In the Single Rule of Fellowship, the Proportion is, as the whole Stock of all the Partners is in Proportion to the total Gain or Loss, so is each Man's particular Share in the Stock, to his particular Share in the Gain or Loss. Therefore take the Total of all the Stocks for the first Term in the Rule of Three, and the whole Gain or Loss for the second Term, and the particular Stock of any one of the Partners for the third Term; then multiply and divide according to the seventh Rule of the ninth Chapter, and the fourth proportional Number is the particular Loss or Gain of him whose Stock you made your

second Number, wherefore repeat the Rule of Three as often as there are particular Stocks or Partners in the Question, and the fourth Terms produced upon the several Operations are the respective Gain or Loss of those particular Stocks given, as in the Examples following.

Quest. 1. Two Persons, viz. A and B bought a Ton of Wine for 20 *l.* of which A paid 12 *l.* and B paid 8 *l.* and they gained in the Sale thereof 5 *l.* now I demand each Man's Share in the Gains, according to his Stock.

First, I find the Sum of all their Stocks, by adding them together, viz. 12 *l.* and 8 *l.* which are 20 *l.* then according to this Rule, I say first, if 20 *l.* (the Sum of their Stocks) require 5 *l.* the total Gain, how much will 12 *l.* (the Stock of A) require? Multiply and divide by the 7th Rule of the 9th Chapter, and the Answer is 3 *l.* for the Share of A in the Gains; then again I say, If 20 *l.* require 5 *l.* what will 8 *l.* require? The Answer is 2 *l.* which is the Gain of B, so I concluded on the Share of A in the Gain is 3 *l.* and the Share of B is the Gain is 2 *l.* which in all is 5 *l.*

$$\begin{array}{r} \text{If } 20 \quad \text{12} \quad \text{8} \\ \quad \quad \quad 5 \quad 12 \\ \quad \quad \quad 12 \end{array}$$

$$\begin{array}{r} 20 \quad 60 \quad (3 \text{ l.}) \\ \quad \quad 60 \end{array}$$

(0)

$$\begin{array}{r} \text{If } 20 \quad \text{12} \quad \text{8} \\ \quad \quad \quad 5 \quad 8 \\ \quad \quad \quad 8 \end{array}$$

$$20 \quad 42 \text{ (l.)}$$

Quest. 2. Three Merchants, viz. A, B, and C, enter upon a joint Adventure, A put into the common Stock 78 *l.* B put in 117 *l.* and C put in 234 *l.* and they find (when they make up their Accompts) that they have gained in all 264 *l.* now I desire to know each Man's particular Share in the Gain.

First, I add their particular Stocks together, and their Sum is 429 l. then say, If 429 l. gain 254 l. what will 78 l. gain? and what 117 l. and what will 234 (the Stocks of A, B, and C) gain? Work by three several Rules of Three, and you will find that

Sum 429

$$\text{The Gain of } \left. \begin{array}{l} A \\ B \\ C \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 48 \\ 72 \\ 144 \end{array} \right.$$

Sum 264

Quest. 3. Four Partners, A, B, C, and D, amongst them built a Ship, which cost 1730 l. of which A paid 346 l. B. 519 l. C 692 l. and D 173 l. and her Freight for a certain Voyage is 370 l. which is due to the Owners or Builders. I demand each Man's Share thereof according to his Charge in Building her.

$$\text{Answer, } \left. \begin{array}{l} A \\ B \\ C \\ D \end{array} \right\} \begin{array}{l} 74 \\ 111 \\ 148 \\ 37 \end{array}$$

Sum 264

Quest. 4. A, B, and C enter into Partnership for a certain Time, A put into the common Stock 364 l. B put in 482 l. C put in 500 l. and they gained 867 l. Now I demand each Man's Share in the Gain, proportionable to his Stock?

$$\text{Answer, } \begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ A \} 234 \quad 09 \quad 3 \\ B \} 310 \quad 09 \quad 5 \\ C \} 32 \quad 00 \quad 3 \end{array}$$

Sum 867 00 0

5. To prove the Single Rule of Fellowship, add each Man's particular Gain or Loss together, [The Proof of the Rule of Single Fellowship] and if the total Sum is equal to the general Gain or Loss, then is the Work rightly performed; but otherwise it is erroneous. Example. In the first Question of this Chapter, the Answer was, That the Gain of A was 3 l. and the Gain of B 2 l. which added together, makes 5 l. equal to the total Gain given.

G 3

If

If in finding out the particular Shares of the several Partners, any Thing remain after Division is ended, such Remainders must be added together, (they being all Fractions of the same Denomination;) and their Sum divided by the common Divisor in each Question, viz. The total Stock) and the Quotient added to the particular Gains, and then if the total Sum is equal to the total Gain, the Work is right, otherwise not.

As in the 4th Question, the Remainders were 354, 62, and 930, which, added together, make 1346, which divided by 1346, the Sum of their Stocks) the Quotient is 1d. which I add to the Pence. 3c. and the Sum of their Share is 897 l. equal to the total Gain, wherefore I conclude the Work is right.

C H A P. XVI.

Double Fellowship.

Double Fellowship, is when several Persons enter into Partnership for unequal Time; that is, when every Man's particular Stock hath Relation to a particular Time.

2. In the Double Rule of Fellowship, multiply each particular Stock by its respective Time, and having added the several Products together, make their Sum the first Number (or Term) in the *Rule of Three*, and the total Gain or Loss the second Number, and the Product of any one's particular Stock by his Time, the third Term, and the fourth Number in Proportion thereunto is his particular Gain or Loss, whose Product of Stock and Time is your third Number.

Then repeat (as in *Single Fellowship*) the *Rule of Three*, as often as there are Products (or Partners) and the four Terms thereby invented, are the Numbers required.

Example.

Quest. 1. A. and B. enter Partnership; A. put in 40 l. for 6 Months, B. put in 75 l. for 4 Months, and they gained 70 l. Now I demand each Man's Share in the Gain, proportional to his Stock and Time? *Answer* A. 20 l. B. 50 l.

1.	1.
40	75
3	4
<hr/>	
A 120	B 300
	120
	<hr/>
Sum	420

G. 4

Then

Then he put in 86 l. which, added to the first (viz. 148 l.) makes 234 l. which lay in Stock 5 Months, and their Product is ——— 1170

Then he put in 100 l. more, so then he had in Stock 334 l. which continued the Remainder of the Time, 4 Months, which multiplied together, produces ——— 1336

The Sum of the Product of the Money and Time of C is ——— 2950

B ——— 4466

C ——— 4688

The total Sum of all the Products is ——— 12104

Then I say, as 12104 is to 1426 (the total Gain) so is 4688 to the Share of A in the total Gain, &c. go on as in the foregoing Examples, and you will find their Shares in the Gain to be as followeth, viz.

Answer

The Share of $\left\{ \begin{array}{l} A \\ B \\ C \end{array} \right\}$ is $\left\{ \begin{array}{ll} 556 & 03 & 6 \\ 529 & 16 & 9 \\ 349 & 19 & 8 \end{array} \right.$

1436 00 0

Quest. 3. Three Grasiers A, B, and C, rake a Piece of Ground for 46 l. 10 s. in which A put 12 Oxen for 8 Months, B put in 16 Oxen for 5 Months, and C put 18 Oxen for 4 Months; now the Question is, what each Man shall pay for the 46 l. 10 s. for his Share in that Charge.

Answer,

$\left. \begin{array}{l} A \\ B \\ C \end{array} \right\}$ shall pay $\left\{ \begin{array}{ll} 18 & 00 \\ 15 & 00 \\ 13 & 10 \end{array} \right.$

46 10

3. The Proof of this Rule is the same with that of Single Fellowship, laid down in the 5th Rule of the 15th Chapter; and note, that

If a Loss be sustained instead of a Gain among Partners, every Man's Share to be born in the Loss, is to be found after the same Method as their Gain, whether their Stock, be for equal or unequal Time.

C H A P. XVII.

Alligation Medial.

1. **T**HE Rule of Alligation is that Rule in Plural Proportion by which we resolve Questions, wherein is a Composition or Mixture of divers Simples; as also it is useful in Composition of Medicines, both for Quantity, Quality, or Price: And its Species are two; viz. Medial and Alternate.

2. Alligation Medial, is, when having the several Quantities and Prices of several Simples propounded, we discover the mean Price or Rate of any Quantity of the Mixture compounded of those Simples, and the Proportion is,

As the Sum of the Simples to be mingled is to the total Value of all the Simples, so is any Part or Quantity of the Composition or Mixture to its mean Rate or Price.

Quest 1. A Farmer mingled 20 Bushels of Wheat at 5 s. per Bushel, and 36 Bushels of Rye at 3 s. per Bushel, with 40 Bushels of Barley at 2 s. per Bushels now I desire to know what one Bushel of that Mixture is worth?

To resolve this Question, add together the given Quantities, and their Value, which is 96 Bushels, whose total Value is 14 l. 8 s. as appeareth by the Work following.

Bushels	l.	s.
20 of Wheat, at 5 s. per Bushel, is	5	0
36 of Rye, at 3 s. per Bushel, is	5	8
40 of Barley, at 2 s. per Bushel, is	4	0
<hr/>		
The Sum of their given Quantities is } 96, and their Value is,	14	8

Then say, by the Rule of Three Direct, if 96 Bushels cost, (or is worth 14 l. 8 s. what is one Bushel worth?

bush.	l.	s.	bush.
96	14	8	1 bush.
	20		
<hr/>			
96)	288	(3 s.	
	.		
	288		
<hr/>			
	(0)		

Facit 3 s. per Bushel.

Quest. 2. A Vintner mingleth 15 Gallons of Canary at 8 s. per Gallon, with 20 Gallons of *Malaga*, at 7 s. 6 d. per Gallon, with 10 Gallons of *Malaga*, at 6 s. 8 d. per Gallon, and 24 Gallons of White-wine at 4 s. per Gallon: Now I demand what a Gallon of this Mixture is worth? Work as in the last Question, and you will find the Answer to be 6 s. 2 d. 2 qrs. $\frac{2}{3}$.

Quest. 3. A Grocer hath mingled 3 C. of Sugar at 56 s. per C. with 3 C. of Sugar at 3 l. 14 s. 8 d. per C. and with 6 C. at 1 l. 17 s. 4 d. per C. I desire to know the Price of a C. wt. of that Mixture.

Answer 2 l. 13 s. 1 d. $\frac{7}{8}$.

3. The Proof of this Operation, is by the Price of any Quantity of the Mixture, to find out the total Value of the whole Composition, and if it is equal to the total Value of the several Simples, the Work is right, otherwise not. [*The Proof of Allegation Medial*]. As in the first Example, the Answer to the Question was, that 3 s. is the Price of 1 Bushel; wherefore I say, by the Rule of Proportion, if 1 Bushel be 3 s. what is 96 Bushels? *Answer*, 14 l. 8 s. which is the total Value of the several Simples: Wherefore the Work is right.

C H A P. XVIII.

Alligation Alternate.

1. **A**lligation Alternate is, when there are given the particular Prices of several Simples, and thereby we discover such Quantities of those Simples, as being mingled together, shall bear a certain Rate propounded.

2. When

2. When such a Question is stated, place the given Prices of the Simples one over the other, and the propounded Price of the Composition against them in such Sort, that it may represent a Root, and they as so many Branches springing from it, as in the following Example.

Quest. 1. A certain Farmer is desirous to mix 20 Bushels of Wheat at 5 s. or 60 d. per Bushel, with Rye at 3 s. or 36 d. per Bushel, and with Barley at 2 s. or 24 d. per Bushel, and Oats at 1 s. 6 d. per Bushel, and desireth to mix such a Quantity of Rye, Barley and Oats, with the 20 Bushels of Wheat, as that the whole Composition may be worth 2 s. 8 d. or 32 d. per Bushel.

The Prices of the Simples being placed according to the last Rule (with the Price of the Composition propounded as a Root to them) will stand as followeth.

$$\begin{array}{r} 60 \text{ Pence} \\ 32 \left\{ \begin{array}{l} 36 \\ 24 \\ 18 \end{array} \right. \end{array}$$

3. Having thus placed the given Numbers, you are to link the several Rates of the Simples one to the other, by certain Arches, in such Sort, that one that is lesser than the mean Rate, may be coupled to another that is greater than the mean Rate; so the Question last propounded will stand

1. Thus,

$$\begin{array}{r} 60 \\ 32 \left\{ \begin{array}{l} 36 \\ 24 \\ 18 \end{array} \right. \end{array}$$

2. Or thus,

$$\begin{array}{r} 60 \\ 32 \left\{ \begin{array}{l} 36 \\ 24 \\ 18 \end{array} \right. \end{array}$$

3. Or thus,

$$\begin{array}{r} 60 \\ 32 \left\{ \begin{array}{l} 36 \\ 24 \\ 18 \end{array} \right. \end{array}$$

4. Then take the Difference between the Root and the several Branches, and place the Difference of each against the Number or Branch with which it is coupled or linked, and having taken all the Differences and placed them as aforesaid, then those Differences so placed, will shew you the Number of each Simple to be taken to make a Composition to bear the mean Rate propounded.

So the Branches of the last Question being linked together, as in the Manner, I say, the Difference between 32 and 60, is 28; which I put against 18, because 60 is linked with 18, then the Difference between 32 and 36 is 4, which I put against 24, because 36 is linked or

coupled with 24, then I say, the Difference between 32 and 24 is 8, which I place against 36, (for the Reason aforesaid) then I say, the Difference between 32 and 18 is 14, which I place against 60; and then the Work will stand as you see in the Margent.

	60	14
	36	8
32	24	4
	18	28

So I conclude that a Composition made of 14 Bushels of Wheat at 6 d. per Bushel, and 8 Bushels of Rye at 36 d. per Bushel, and 5 Bushels of Barley at 24 d. per Bushel, and 28 Bushels of Oats at 18 d. per Bushel, will bear the mean Price of 32 d. or 2 s. 8 d. per Bushel. And here observe, That in the Composition there is but 14 Bushels of Wheat; but I would mingle 20 Bushels, and this Kind (or rather Case) of Alligation Alternate, (*viz.*) when there is given a certain Quantity of one of the Simples, and the Quantities of the rest sought to mingle with this given Quantity, (that the whole may bear a Price propounded) is called Alternation Partial.

And the Proportion to find out the several Quantities to be mingled with the given Quantity, is thus.

As the Difference annexed to the Branch, that is, the Value of an Integer of the given Quantity is to the other particular Differences, so is the Quantity given to the several Quantities required.

So here, to find how much Rye, Barley, and Oats, must be mingled with the 20 Bushels of Wheat, I say, by the Rule of Three direct, if 14 Bushels of Wheat require 8 Bushels of Rye, what will 20 Bushels of Wheat require?

Answer, $11\frac{6}{7}$ Bushels of Rye.

Again, If 14 Bushels of Wheat require 4 Bushels of Barley, what will 20 Bushels of Wheat require? *Ans.* $5\frac{1}{2}$ Bushels of Barley. Again, I say, if 14 Bushels of Wheat require 28 Bushels of Oats, what will 20 Bushels of Wheat require? *Answer,* 40 Bushels of Oats.

And now I say, that 20 Bushels of Wheat mingled with $11\frac{6}{7}$ Bushels of Rye, and $5\frac{1}{2}$ Bushels of Barley, and 40 Bushels of Oats, each bearing the Rate as aforesaid, will make a Composition or Heap of Corn, that may yield 32 d. per Bushel.

But if the Branches had been coupled according to the second Order, or Manner, the Differences would have been thus placed, *viz.* the Difference between 33 and 60 is 28, which I set against 24, because 60 is linked thereto; and the Difference between 32 and 36 is 4, which I set against 18; and the Difference between 32 and 24 is 18, which I set against 60; then the Difference between 32 and 18 is 14, which I set against his Yoke-fellow 36; and then I conclude, that if you mixt 8 Bushels of Wheat with 14 Bushels of Rye, 28 Bushels of Barley, and 4 Bushels of Oats, each bearing the aforesaid Prices, the whole Mixture may be sold for 32 *d.* per Bushel, as by the Work in the Margent.

32	{	60	}	8
		36		14
		24		28
		18		4

You see by this Work we have found how many Bushels of Rye, Barley, and Oats, ought to be mixed with 8 Bushels of Wheat, and to find out how many of each ought to be mixed with 20 Bushels of Wheat, I say, as 1 is to 14, so is 20 to 35 Bushels of Rye. As 8 is to 28, so is 20 to 70 Bushels of Barley. As 8 is to 4, so is 20 to 10 Bushels of Oats; whereby I conclude, that if to 20 Bushels of Wheat I put 35 Bushels of Rye, 70 Bushels of Barley, and 10 Bushels of Oats, bearing each the aforesaid Price per Bushel, that then a Bushel of this Mixture will be worth 32 *d.* or 2 *s.* 8 *d.*

And if the Branches had been linked, as you see in the 3d Place, where each Branch bigger than the Root is link'd to two that are lesser than the Root, then in this Case you must have placed the several Differences between the Root and Branches, against those two with which each is coupl'd, as first, the Difference between 32 and 60 is 28, which I set against 24 and 18, because it is coupled with them

32	{	60	}	8	14	22
		36		8	14	22
		24		28	4	32
		18		28	4	32

both; then the Difference between 32 and 36 is 4, which I set likewise against 3, and 18, because 36 is linked to them both, then the Difference between 32 and 24 is 8, which I put against 60 and 36, because 24 is linked to them.

them both, then the Difference between 32 and 18 is 14, which I put against 60 and 36, the Yoke-Fellows of 18.

Lastly, I draw a Line behind the Differences, and add the Differences which stand against each Branch, and put the Sum behind the said Line against its proper Branch, as you see in the Margent.

And now by this Work, I find that 22 Bushels of Wheat mingled with 22 Bushels of Rye, and 32 Bushels of Barley, and 32 Bushels of Oats, each bearing the said Price, will make a Mixture bearing the mean Rate of 32 *d. per Bushel*.

And to find how much of each of the rest must be mingled with 20 Bushels of Wheat, I say,

As 22 is to 32, so is 20 to 29 Bushels of Rye. As 22 is to 32, so is 20 to $19\frac{1}{2}$ Bushels of Barley. As 22 is to 32, so is 20 to $29\frac{1}{2}$ Bushels of Oats.

Whereby you see the Questions of Alligation Alternate, will admit of more true Answers than one; for we have found 3 several Answers to this 1st Question.

The Proof of Alternation Partial.

Questions of Alligation Partial are proved the same Way with Questions in Alligation Medial, which you may see in the 3d Rule of the 17th Chapter.

Quest. 3. A Grocer hath 4 Sorts of Sugar, viz. of 12 *d. per l.* of 10 *d. per l.* of 6 *d. per l.* and of 4 *d. per l.* and would have a Composition worth 8 *d. per l.* the whole Quantity whereof should contain 144 *l.* made of these four Sorts, I demand how much of each he must take.

Questions of this Nature are resolved by that Part of Alligation Alternate, called by Arithmeticians Alligation Total, viz. where there is given the Sum and Prices of several Simples, to find out how much of each Simple ought to be taken to make the said Sum or Quantity, so that it may bear a certain Rate propounded.

To resolve this Question, I place the several Prices of the Simples and Mean Rate propounded, and link them together, as is directed in the 2d and 3d Rules of this Chapter, and place the Differences between the Root and Branches, according to the 4th Rule of this Chapter, which will then stand one of these 3 Ways, viz.

First

$$\begin{array}{r}
 8 \left\{ \begin{array}{l} 12 \\ 10 \\ 6 \\ 4 \end{array} \right. \quad \left| \begin{array}{l} 4 \\ 2 \\ 2 \\ 4 \end{array} \right. \\
 \hline
 12
 \end{array}$$

Second.

$$\begin{array}{r}
 8 \left\{ \begin{array}{l} 12 \\ 10 \\ 6 \\ 4 \end{array} \right. \quad \left| \begin{array}{l} 2 \\ 4 \\ 4 \\ 2 \end{array} \right. \\
 \hline
 12
 \end{array}$$

Third.

$$\begin{array}{r}
 8 \left\{ \begin{array}{l} 12 \\ 10 \\ 6 \\ 4 \end{array} \right. \quad \left| \begin{array}{l} 2, 4 \\ 2, 4 \\ 4, 2 \\ 4, 2 \end{array} \right. \quad \left| \begin{array}{l} 6 \\ 6 \\ 6 \\ 6 \end{array} \right. \\
 \hline
 24
 \end{array}$$

5. Then add the several Differences together, which I have done, and the Sums of the first and second Order are 12 l. and of the 3d, 24 l. as you see above. But it required that there should be 144 l. of the Composition, therefore to find the Quantity of each Simple to make the whole Composition 144 l. Observe this general Rule, viz.

As the Sum of the Differences is to the several Differences, so is the total Quantity of the Composition to the Quantity of each Simple.

So to find how much of each Sort of Sugar I ought to take to make 144 l. at 8 d. per l.

As 12 is to 4, so is 144 to 48 l. at 12 d. per l.

As 12 is to 2, so is 144 to 24 l. at 10 d. per l.

As 12 is to 2, so is 144 to 24 l. at 6 d. per l.

As 12 is to 4, so is 144 to 48 l. at 4 d. per l.

Whereby I find that 48 l. at 12 d. per l. and 24 l. at 10 d. per l. and 24 l. at 6 d. per l. and 48 l. at 4 d. per l. will make a Composition of Sugar containing 144 l. worth 8 d. per l.

But as the Branches are linked in the 2d Order, the Answer will be 24 l. at 12 d. per l. and 48 l. at 10 d. per l. and 48 l. at 6 d. per l. and 24 l. at 4 d. per l. to make the said Quantity, and to bear the said Price.

And if you had work'd as the Branches are linked after the third Order, then you would have found the Quantity of 36 l. of each.

Quest. 3. A Vintner hath four sorts of Wine, viz. Canary

at 10 s. per Gallon, Malaga at 8 s. per Gallon, Rhenish-wine at 6 s. per Gallon, and White-wine at 4 s. per Gallon, and he is minded to make a Composition of them all of 60 Gallons, that they may be worth 5 s. per Gallon, I desire to know how much of each he must have?

The Number of Terms being ranked according to the second Rule of this Chapter, the Branches will be linked as followeth; but will admit of no other manner of coupling, because there is but one Branch that is lesser than the Root; therefore all the rest must be linked unto it; and the Differences between the Root and the three first Branches, viz. 10, 8, and 6, which are 5, 3, and 1, must be set against 4, because they are coupled with it, and the Difference between the Root, (viz.) 5 and 4, which is 1, must be set against the 3 other, because it is linked to them all; so I find 1 Gallon of Canary, 1 Gallon of Malaga, 1 Gallon of Rhenish wine, and 9 Gallon of White-wine, prized as above, being mingled together, will be worth 5 s. per Gallon, the Sum being 12 Gallons; but there must be 60 Gallons; whereof I say,

As 12 is to 1, so is 60 to 5 Gallons of Canary.

As 12 is to 1, so is 60 to 5 Gallons of Malaga.

As 12 is to 1, so is 60 to 5 Gallons of Rhenish.

As 12 is to 9, so is 60 to 45 Cal. of White-wine.

so that 5 Gallons of Canary, 5 Gallons of Malaga, 5 Gallons of Rhenish; and 45 of White wine, mingled together, will be in all 60 Gallons worth 5 s. per Gallon, which was required.

Quest. 4. A Goldsmith hath Gold of four several Sorts of Fineness, viz. of 24 Carects fine, and of 22 Carects fine, of 20 Carects fine, and of 15 Carects fine. [*Read Chap. 2. Def. 2. of this Book.*] And he would mingle so much of each with Allay, that the whole Mass of 28 Ounces of Go'd so mingled; may bear 17 Carects fine. I demand how much of each he must take? The 2d and 3d Rules of this Chapter being observed; (for instead of the Allay I put 0, because it bears no Fineness, but it makes a Branch in the Operation) the Terms may be alligated, and the Differences added by any of these 4 Ways following, viz.

10	1	1
8	1	1
6	1	1
4	5, 3, 1.	0

First,

First thus,

17	{	2		17		17
		22		2		2
		20		1, 17		19
		15		5, 3		8
		9		7, 3		10

Sum 56

Secondly thus,

17	{	24		2		2
		22		17		17
		20		2, 17		19
		15		7, 3		10
		0		5, 3		8

Sum 56

Thirdly thus,

17	{	24		3		2
		22		2		2
		20		2, 17		19
		15		7, 5, 3		15
		0		3,		3

Sum 41

Fourthly thus,

17	{	24		2,	17,	19
		2		2,	17,	19
		20		2,	17,	19
		15		7, 5,	3,	15
		0		7, 5,	3,	19

Sum 87

More Ways may be given for the alligating or linking of the Terms in this Question, but these, if well practised, are sufficient for understanding the Rules of Alligation

In Questions of Alligation Total the Answer is given true, when the Sum of each of the Quantities of Simples found, [*The Proof of Alternation Total*] agrees with the Sum or Quantity propounded; as in the last Question, the Answer was 8 oz. 10 p. w. of 24 Carects fine, 10 oz. of 22 Carects fine, 9 oz. 10 p. w. of 20 Carects fine, 4 of

25 Carets fine, and 5 oz. of Alloy, which added together, make 28 oz. the Quantity propounded.

C H A P. XIX.

Reduction of Vulgar Fractions.

1. **W**HAT a Vulgar Fraction is, hath been already shewed, in the first Chapter of this Book, to which I refer the Reader to look cautiously into.

2. To reduce a Vulgar Fraction, observe carefully these 8 following Rules.

1. To reduce a mixt Number into an improper Fraction.

2. To reduce a whole Number into an improper Fraction.

3. To reduce an improper Fraction into its equivalent Whole, (or Mixt) Number.

4. To reduce a Fraction into the lowest Terms equivalent to the Fraction given.

5. To find the Value of a Fraction in the known Parts of Coin, Weight, Measure, &c.

6. To reduce a Compound Fraction to a simple one of the same Value.

7. To reduce divers Fractions having unequal Denominations, to Fractions of the same Value, having an equal Denominator.

8. To reduce a Fraction of one Denomination to another of the same Value.

I. To reduce a mixt Number to an improper Fraction.

The Rule is,

Multiply the Integer Part (or whole Number) by the Denominator of the Fraction [*Vide Chap. I. Defin. 31.*] and to the Product add the Numerator, and that Sum place over the Denominator for a new Numerator, so this new Fraction shall be equal to the mixt Number given. As for Example.

1. Reduce $18 \frac{3}{7}$ into an improper Fraction, multiply the whole Number 18 by 7 the Denominator, and to the Product add the Numerator 3, the Sum is 129, which put over the Denominator 7, and it makes $18 \frac{3}{7}$ for the Answer as followeth.

$$18 \frac{3}{7}$$

$$\frac{7}{7}$$

Facit 129

$$\frac{7}{7}$$

2. Reduce $113 \frac{1}{2}$ to an improper Fraction, *facit*, $226 \frac{1}{2}$

3. Reduce $50 \frac{1}{2}$ to an improper Fraction, *facit*, $101 \frac{1}{2}$

II. To reduce a Whole Number into an improper Fraction.

The Rule is, Multiply the given Number by the intended Denominator, and place the Product for the Numerator over it. [Vide Chap. 1. Defn. 23.] As for Example.

1. Let it be required to reduce 15 into a Fraction, whose Denominator shall be 12. To effect which, I multiply 15 by the intended Denominator (12) the Product is 180, which I place over 12 as a Numerator, and it makes $15 \frac{0}{12}$, which is equal to 15 *Facit* $15 \frac{0}{12}$
as was required; as per Margent.

2. Reduce 36 into an improper Fraction, whose Denominator shall be 26, *Facit* $9 \frac{6}{13}$.

3. Reduce 135 into an improper Fraction, whose Denominator shall be 16, *Facit* $21 \frac{9}{16}$.

III. To reduce an improper Fraction into its equivalent whole or mixt Number.

The Rule is; Divide the Numerator by the Denominator, and the Quotient is the whole Number equal to the Fraction, and if any Thing remain, put it for a Numerator over the Divisor. *Example*,

1. Reduce $43 \frac{6}{8}$ into its equivalent mixt Number. Divide the Numerator 436 by the Denominator 8, and the Quotient is 54, and 3 remains, which put for a Numerator over the Divisor 8, the Answer is $54 \frac{3}{8}$, as followeth,

$$8) 436 (54$$

$$\underline{40}$$

$$36 \text{ Facit } 54 \frac{3}{8}$$

$$\underline{32}$$

$$(4)$$

2. Reduce $14\frac{7}{8}$ to a mixt Number, *Facit* $231\frac{1}{8}$.
 3. Reduce $25\frac{7}{8}$ to a mixt Number, *Facit* $114\frac{1}{8}$.

IV. To reduce a Fraction into its lowest Terms equivalent to the Fraction given.

The Rule is, 1. If the Numerator and Denominator are even Numbers, take half the one and half of the other, as often as may be, and when either of them falls out to be an odd Number, then divide them by any Number that you can discover will divide both Numerator and Denominator without any Remainder; and when you have thus proceeded as low as you can reduce them, then this new Fraction so found out, shall be the Fraction you desire, and will be in Value equal to the given Fraction.

Example. 1. Let it be required to reduce $\frac{129}{336}$ into its lowest Terms. First I take the Half of the Numerator 129, and it is 96, then Half of the Denominator, and it is 168, so that it is brought to $\frac{96}{168}$, and next to $\frac{48}{84}$, and by halving still, to $\frac{24}{42}$ and their half is $\frac{12}{21}$, and now I can no longer half it, because 21 is an odd Number, wherefore I try to divide them by 3, 4, 5, 6, &c. and I find 3 divides them both without any Remainder, and brings 'em to $\frac{4}{7}$, as per Margent.

So I conclude $\frac{4}{7}$ thus found to be equal in Value to the given Fraction $\frac{129}{336}$.

2. What is $\frac{1016}{1794}$ in its lowest Terms? *Ans.* $\frac{7}{8}$.

3. What is $\frac{1342}{1794}$ in its lowest Terms? *Ans.* $\frac{1}{3}$.

The best Way to reduce a Fraction into its lowest Terms, is, by finding a common Measurer, viz. the greatest Number that will divide the Numerator and Denominator without any Remainder, and by that Means reduce a Fraction to its lowest Terms at the first Work; and to find out this common Measurer, divide the Denominator by the Numerator, and if any thing remains, divide your Divisor thereby; and if any thing yet remain, then divide your last Divisor by it, do so until you find nothing remaining: Then this last Divisor shall be your greatest common Measurer, which will divide both Numerator and Denominator, and reduce 'em both into their lowest Terms at one Work.

Ex

Example. 4. Reduce $\frac{228}{304}$ into its lowest Terms by a common Measurer; to Effect which, I divide the Denominator 304 by the Numerator 228, and there remains 76, then I divide 228 (the first Divisor) by 76 (the Remainder) and it quotes 3, and nothing remains; wherefore the last Divisor 76 is the common Measurer; by which I divide the Numerator of the given Fraction, viz. 228 it quotes 3 for a new Numerator, then I divide the Denominator 304 by 76, and it quotes 4 for a new Denominator, so that now I have found $\frac{3}{4}$ equal to $\frac{228}{304}$.

5. Reduce $\frac{604}{729}$ into its lowest Terms by a common Measurer, *Facit*, $\frac{2}{3}$.

6. Reduce $\frac{3081}{1052}$ into its lowest Terms by a common Measurer, *Facit*, $\frac{1}{2}$.

A Compendium.

Note, That if the Numerator or Denominator of a Fraction, end each with a Cypher or Cyphers, then cut off as many Cyphers from the one as from the other, and the remaining Figures will be a Fraction of the same Value, viz. $\frac{3400}{7100}$ will be found to be reduced to $\frac{34}{71}$, by cutting off the two Cyphers from the Numerator and Denominator with a Dash of the Pen thus, $\frac{34}{71} \frac{00}{00}$, and $\frac{460}{700}$ will be $\frac{46}{70}$, thus, $\frac{46}{70} \frac{0}{0}$, &c.

V. To find the Value of a Fraction in the known Parts of Coin, Weights, &c.

The Rule is, Multiply the Numerator by the Parts of the next inferior Denomination that are equal to an Unit of the same Denomination with the Fraction; then divide that Product by the Denominator, and the Quote gives you its Value in the same Parts you multiply'd by, and if any Thing remain, multiply it by the Parts of the next inferior Denomination, and divide as before; do so till you can bring it no lower, and the several Quotients, will give you the Value of the Fraction as was required: and if any at last remain, place it for a Numerator over the former Denominator. Some few Examples will make the Rule plain.

L. What

1. What is the Value of $\frac{27}{29}$ l. Sterling? To answer this Question, I multiply the Numerator 27 by 20, (the Shillings in a Pound) the Product is 540, which I divide by 29 (the Denominator) and the Quotient is 18 s. and there remains 18, which I multiply by 12 Pence, and the Product (216) I divide by the Denominator 29, the Quotient is 7 d. and 13 remains, which I multiply by 4 Farthings, the Product is 52, which I still divide by 29, the Quotient is 1 qr. and there remaineth 23, which I put for a Numerator over the Denominator 29, so I find the Value of $\frac{27}{29}$ l. to be 18 s. 7 d. 1 qr. $\frac{23}{29}$, as by the Work in the Margent, and after the same Manner are the Values of the Fractions in the several Examples following found out.

	$\frac{27}{29}$ l.
	27
Multiply	29
	<hr/>
	29) 540 18 s. 7 d. $1\frac{23}{29}$ qr.
	..
	<hr/>
	29
	<hr/>
	250
	232
	<hr/>
Rem.	(18)
Mult.	12)
	<hr/>
	36
	18
	<hr/>
	29) 216 7 d.
	203
	<hr/>
Rem.	(13)
Mult.	4
	<hr/>
	qr.
	29) 52 ($1\frac{23}{29}$
	29
	<hr/>
Rem.	(23)
	s. d. qr.
	Facit 18 7 $1\frac{23}{29}$

And so likewise you may find the Value of any Fraction either in Weight or Time, &c.

VI. To reduce a Compound Fraction to a Simple of the same Value.

What a Compound Fraction is, hath been shewn in Chap. 1. Definition 24, and to reduce it to a Simple Fraction of the same Value.

The Rule is, Multiply the Numerators continually, and place the last Product for a new Numerator, then multiply the Denominators continually, and place the last Product for a new Denominator. So this single Fraction shall be equal to the compound Fraction. Example.

1. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a simple Fraction.

Multiply the Numerators 2, 3, and 5 together, they make 30 for a new Numerator; then I multiply the Denominators 3, 5, and 8 together, and their Product is 120 for a Denominator, so the simple Fraction is $\frac{15}{120}$, and cutting off the Cyphers, it is $\frac{1}{8}$ equal to $\frac{1}{8}$ by the 4th Rule following.

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 8 \end{array}$$

120

$$\begin{array}{r} 3 \\ 2 \\ \hline 6 \\ 5 \end{array}$$

30



Facit $\frac{1}{8}$, or $\frac{1}{8}$, or $\frac{1}{8}$.

2. What is $\frac{1}{7}$ of $\frac{5}{8}$ of $\frac{4}{9}$ of $\frac{1}{12}$? *Answer* $\frac{5}{756}$, or $\frac{1}{151.2}$ or $\frac{7}{756}$ in its lowest Terms.

3. What is $\frac{1}{12}$ of $\frac{3}{4}$ of $\frac{2}{9}$? *Answer* $\frac{1}{54}$.

By this you may know how to find the Value of a compound Fraction, viz. First reduce it to a Simple one, and then find out his Value by the 5th Rule foregoing.

Example. 4. What is the Value of $\frac{3}{4}$ of $\frac{2}{5}$ of $\frac{1}{2}$ of a Pound? *Answer* 11 s. 3 d.

VII. To reduce Fractions of unequal Denominators to Fractions of the same Value, having equal Denominators.

The Rule is, Multiply all the Denominators together, and the Product shall be the common Denominator. Then multiply each Numerator into all the Denominators, except its own, and the last Product put for a Numerator over the Denominator, found out as before: So this new Fraction is equal to that Fraction whose Numerator you multiply into the said Denominators. Do so by all the Numerators given, and you have your Desire.

Example. 1. Reduce $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ and $\frac{7}{8}$ to a common Denominator. Multiply the Denominators 4, 5, 6, and 8 together continually, and put the Products 960 for the common Denominator; then multiply the Numerator 3 into the Denominators 5, 6, and 8, and the Product is 720, which is a Numerator to 960 (found as before) so $\frac{720}{960}$ is equal to the first Fraction $\frac{3}{4}$; then I proceed to find a new

new Numerator to the 2d Fraction, viz. $\frac{4}{3}$, and I multiply 4 (into all the Denominators except its own, viz.) into 4, 6, and 8, which produceth $\frac{1}{2} \times \frac{8}{8}$ equal to $\frac{4}{2}$, then multiply the Numerator 5 into the Denominators 4, 5, and 8, the Product is $\frac{800}{2}$ equal to $\frac{5}{2}$. Then multiply the Numerator 7 into the Denominators 4, 5, and 6, the Product is $\frac{840}{2}$ equal to $\frac{7}{2}$, and the Work is done; so that for $\frac{3}{4} \times \frac{5}{8}$ and $\frac{7}{8}$, I have $\frac{730}{2}$, $\frac{668}{2}$, $\frac{800}{2}$ and $\frac{840}{2}$.

2. Reduce $\frac{11}{12}$, $\frac{14}{25}$, and $\frac{19}{21}$ in a common Denominator, *faciunt*, $\frac{3311}{7500}$, $\frac{3728}{7500}$, and $\frac{5244}{7500}$.

VIII. To reduce a Fraction of one Denomination to another.

1. This either Ascending or Descending. Ascending when a Fraction of a smaller is brought to a greater Denomination; Descending, when a Fraction of a greater Denomination is brought lower.

2. When a Fraction is to be brought from a lesser to a greater Denomination, then make of it a Compound Fraction, by comparing it with the intermediate Denominations between it, and that you would have it reduced to then (by the 6th Rule foregoing) reduce your Compound to a single Fraction, and the Work is done. *Example.*

Quest. 1. It is required to know what Part of a Pound Sterling $\frac{1}{2}$ of a Penny is?

To resolve this, I consider that 1 *d.* is $\frac{1}{2}$ of a Shilling, and a Shilling is $\frac{1}{20}$ of a Pound; wherefore $\frac{1}{2}$ *d.* is $\frac{1}{40}$ of $\frac{1}{20}$ of a Pound, which by the said 6th Rule I find to be $\frac{1}{800}$ of a Pound *Sterl. of English Money.*

Quest. 2. What Part of a Pound *Troy-weight* is $\frac{1}{2}$ of a Penny-weight? *Ans.* $\frac{1}{4}$ of $\frac{1}{20}$ of $\frac{1}{12}$, equal to $\frac{1}{2400}$ *Troy.*

3. When a Fraction is to be brought from a greater to a lesser Denomination, then multiply the Numerator by the Parts contained in the several Denominations betwixt it, and the Parts you would reduce it to; then place the last Product over the Denomination of the given Fraction. *Exam.*

Quest. 3. I would reduce $\frac{3}{5}$ *l.* to the Fraction of 1 *d.* to do which, I multiply the Numerator 3 by 20 and 12, the Product is 720, which I put over the Denominator 5, it makes $\frac{720}{5}$ of 1 *d.* equal to $\frac{144}{1}$.

Quest. 4. What Part of an Ounce *Troy* is $\frac{1}{2}$? *Answer,* $\frac{1}{10}$ *oz.*

CHAP. XX.

Addition of Vulgar Fractions.

1. IF your Fractions to be added have a common Denominator, then add all the Numerators together, and place their Sum for a Numerator to the common Denominator, which new Fraction is the Sum of all the given Fractions; and if it be improper, reduce it to a whole or mixt Number, by the 3d Rule in the 29th Chapter.

Quest. 1. What is the Sum of $\frac{7}{24}$, $\frac{9}{24}$, $\frac{16}{24}$, and $\frac{14}{24}$?

The Denominators are equal, viz. every one is 24, wherefore add the Numerators together, viz. 7, 9, 16, and 14, their Sum is 46, which put over the Denominator 24, it makes $\frac{46}{24}$ the Sum of the given Fractions, which will be reduced to the mixt Numbers $1\frac{23}{12}$, or $1\frac{1}{2}$.

2. But if the Fractions to be added have unequal Denominators, then reduce them to a common Denominator by the 7th Rule of Chapter 19. and then add the Numerators together, and put the Sum over the common Denominator, &c. as before in the last Example.

Quest. 2. What is the Sum of $\frac{1}{3}$, $\frac{2}{8}$, $\frac{1}{4}$, and $\frac{1}{12}$?

The Fractions reduced to a common Denominator, are $\frac{4}{12}$, $\frac{3}{12}$, $\frac{3}{12}$, and $\frac{1}{12}$, the Sum of their Numerators is 11, which put over the common Denominator, makes $\frac{11}{12}$ equal to the mixt Numbers $3\frac{1}{4}$, or $3\frac{1}{2}$, for the Sum required.

Quest. 3. What is the Sum of $\frac{1}{7}$, $\frac{1}{5}$, and $\frac{2}{7}$?

Answer. $1\frac{7}{35}$.

3. If you are to add mixt Numbers together, then add the Fractional Parts as before, and if their Sum be an improper Fraction, reduce it to a mixt Number, and add its integral Part to the integral Parts of the given mixt Numbers, and the Work is done.

Quest. 4. What is the Sum of $13\frac{3}{4}$ and $24\frac{5}{8}$?

First add the Fractions $\frac{3}{4}$ and $\frac{5}{8}$, the Sum is $\frac{11}{8}$, then add the Integer 1 to 13 and 24, their Sum is 38, and put after it the Fraction $\frac{11}{8}$ it is $3\frac{11}{8}$ for the Answer, or it is $38\frac{3}{4}$.

Quest. 5. What is the Sum of $48\frac{3}{7}$, $64\frac{5}{8}$, and $13\frac{3}{4}$?

Facit $243\frac{11}{56}$, or $243\frac{1}{5}$.

4. If any of the Fractions to be added, is a Compound Fraction, it must first be reduced to a Simple Fraction by

H

the

the sixth Rule of Chapter 19, and then add it to the rest, according to the second Rule of this Chapter. *Example.*

Quest. 6. What is the Sum of $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{8}$?

Reduce $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{8}$ into a simple Fraction, and it is $\frac{105}{128}$, which reduced with the other two, and added, are $1\frac{137}{128}$.

Quest. 7. What is the Sum of $\frac{1}{2}$ and $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{8}$?

Answer. $1\frac{5}{8}$.

5. If the Fractions to be added are not of one Denomination, they must be so reduced, and then proceed as before.

Quest. 8. What is the Sum of $\frac{3}{4}$ l. and $\frac{5}{8}$ s.

Of the given Fractions here, one is of a Pound, and the other the Fraction of a Shilling; and before you can add them together, you must reduce $\frac{5}{8}$ s. to the Fraction of a Pound as the other is (by the 8th Rule of Chap. 19.) and it makes $\frac{5}{16}$ l., then $\frac{3}{4}$ and $\frac{5}{16}$ l. will be found to be $\frac{13}{8}$ l. by the 7th Rule of Chap. 19. and in its lowest Terms $1\frac{5}{8}$ l. by the 4th Rule of Chap. 19.

It would have been the same if (by the latter Part of the 8th Rule of Chapter 19.) you had reduced $\frac{3}{4}$ l. to the Fraction of a Shilling; which you would have found to have been $\frac{9}{4}$ s. which added to $\frac{5}{8}$ s. by the said 17th Rule of the last Chapter, the Sum is 15 s. $\frac{2}{4}$, which is equal to the Sum found, as before, viz. $1\frac{5}{8}$ l. for (by the 5th Rule of Chapter 19.) the Value of $\frac{13}{8}$ l. will be found to be 15 s. 10 d. and so will 15 s. $\frac{2}{4}$ be found to be just as much.

Quest. 9. What is the Sum of $\frac{3}{4}$ l. $\frac{3}{4}$ s. and $\frac{3}{4}$ d?

Answer. $\frac{379}{640}$ or $\frac{379}{640}$ l. or in its lowest Terms $\frac{379}{640}$.

CHAP. XXI.

Subtraction of Vulgar Fractions.

1. **T**HE Rules in Addition for reducing the given Fractions to one Denomination, are here to be observed; for before Subtraction can be made, the Fractions must be reduced to a common Denominator, then subtract one Numerator from the other, and place the Remainder over a common Denominator, which Fraction shall be the

Ex-

Excess or Difference between the given Fractions. Exam-
ples.

Quest. 1. What is the Difference between $\frac{3}{4}$ and $\frac{1}{8}$? The given Fractions are reduced to $\frac{6}{8}$, and $\frac{1}{8}$, then subtract the Numerator 20 from the Numerator 21, and there remains 1, which being put over the Denominator 28, makes $\frac{1}{28}$ for the Answer or Difference between $\frac{3}{4}$ and $\frac{1}{8}$.

Quest. 2. What is the Difference between $\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{3}{4}$? Reduce the Compound Fraction $\frac{1}{3}$ of $\frac{3}{4}$ to a simple Fraction, then proceed as before, and the Answer is $\frac{1}{4}$ equal to $\frac{1}{4}$.

2. When a Fraction is given to be subtracted from a whole Number, subtract the Numerator from the Denominator, and put the Remainder for a Numerator to the given Denominator, and subtract an unite (for that you borrowed) for the whole Number, and the Remainder place before the Fraction found, as before, which mixed Number is the Remainder or Difference sought. Example.

Quest. 3. Subtract $\frac{7}{10}$ from 48.

Answer. $37\frac{3}{10}$; for if you subtract 7 (the Numerator) from 10 (the Denominator) there remains 3, which put over is $10\frac{3}{10}$, and 1 (1 borrowed) from 48 rests 47, to which join $\frac{3}{10}$, and it makes $47\frac{3}{10}$ for the Excess.

Quest. 4. Subtract $\frac{1}{11}$ from 57, remain $56\frac{10}{11}$.

3. If it be required to subtract a Fraction from a mixt Number, or one mixt Number from another, reduce the Fractions to a common Denominator, and if the Fraction to be subtracted be lesser than the other, then subtract the lesser Numerator from the greater, and that is a Numerator for the common Denominator; then subtract the lesser integral Part from the greater, and the Remainder with the remaining Fractions thereunto annexed, is the Difference required between the two given mixt Numbers. Example.

Quest. 5. Subtract $26\frac{3}{4}$ from $54\frac{5}{8}$.

First, Subtract $\frac{3}{4}$, viz. $\frac{6}{8}$ from $\frac{5}{8}$, viz. $\frac{5}{8}$, the Remainder is $\frac{1}{8}$, then 26 from 54, remaineth 28, to which annex $\frac{1}{8}$ it makes $28\frac{1}{8}$ for the Answer.

4. But if the Fraction to be subtracted is greater than the Fraction from whence you subtract, then having first reduced the Fractions to a common Denominator, take the Numerator of the greatest Fraction out of the Denominator,

and add the Remainder to the Numerator of the lesser Fraction, and their Sum is a new Numerator to the common Denominator, which Fraction Note, then (for the one you borrowed) add one to the integral part to be subtracted, and subtract it from the greater Number, and to the Remainder annex the Fraction you noted before, so this new mixt Number shall be the Difference sought. *Example.*

Quest. 6. Subtract $14\frac{1}{2}$ from $29\frac{4}{7}$.

The Fraction, reduced are, viz. $\frac{3}{4}$ equal to $\frac{21}{28}$, and $\frac{4}{7}$ equal to $\frac{16}{28}$, now I should subtract $\frac{21}{28}$ from $\frac{16}{28}$, but I cannot, therefore I subtract 21 from 28, rest 7, which added to 16 (the lesser Numerator) make 23 for a Numerator to 28, viz. $\frac{23}{28}$; then I come to the integral Parts 14 and 19, and say, 1 that I borrowed and 14 is 15. which taken from 29, there rests 14, to which annexing $\frac{23}{28}$ it is $14\frac{23}{28}$, for the Remainder or Difference between $14\frac{1}{2}$ and $29\frac{4}{7}$.

Quest. 7. Subtract $36\frac{2}{5}$ from $74\frac{4}{5}$? *Facit*, $37\frac{2}{5}$.

CHAP. XXII.

Multiplication of Vulgar Fractions.

1. IF the Multiplicand and Multiplier are simple Fractions, then multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator, and the new Fraction is the Product required.

Quest. 1. What is the Product of $\frac{5}{7}$ by $1\frac{9}{11}$? *Facit* $\frac{45}{77}$, for the Numerators 5 and 9 being multiply'd, make 45, and the Denominators 7 and 11 being multiply'd make 77.

Quest. 2. What is the Product of $\frac{8}{5}$ by $\frac{3}{7}$? *Facit* $\frac{24}{35}$.

2. If the Fractions to be multiplied be mixt Numbers, reduce them to improper Fractions by the first Rule of the 19th Chapter: then proceed as before.

Quest. 3. What is the Product of $28\frac{1}{2}$ by $13\frac{5}{8}$?

The given mixt Numbers being reduced to improper Fractions are $48\frac{1}{2}$ equal to $97\frac{1}{2}$, and $13\frac{5}{8}$ equal to $109\frac{5}{8}$, now $97\frac{1}{2}$ multiplied by $109\frac{5}{8}$, according to the first Rule of this Chapter, produceth $10659\frac{5}{8}$, or $672\frac{3}{4}$.

Quest. 4. What is the Product of $430\frac{6}{11}$ by $18\frac{3}{7}$? *Facit* $7740\frac{4}{11}$, or $703\frac{4}{11}$.

3. If a Compound Fraction is to be multiplied by a Simple

ple Fraction, first reduce the Compound Fraction into a Simple Fraction, then multiply the one by the other, as is taught above.

Quest. 5. What is the Product of $\frac{1}{2}$ of $\frac{1}{3}$ by $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$?

The Compound Fraction $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ reduced is $\frac{1}{24}$, or $\frac{1}{24}$ which multiply by $\frac{1}{5}$ produceth $\frac{1}{120}$, which in its lowest Term is $\frac{1}{120}$ for the Answer.

And if the Multiplicand and Multiplier are both Compound Fractions, reduce them both to Simple ones, then multiply these new Fractions as before, so you have the Product.

Quest. 6. What is the Product of $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$?

Answer. $\frac{1}{24}$ in its lowest Terms.

Quest. 7. What is the Product of $\frac{1}{2}$ of $\frac{1}{3}$ by $\frac{1}{4}$ of $\frac{1}{5}$?

Answer. $\frac{1}{120}$, or $\frac{1}{120}$, or in its least Terms $\frac{1}{120}$.

4. If a Fraction be to be multiplied by a whole Number, put under the given whole Number an Unit for a Denominator, whereby it will be an improper Fraction, then multiply the Fraction as before. *Example.*

Quest. 8. What is the Product of 24 by $\frac{1}{3}$?

Answer. $4\frac{2}{3}$, for 24 by putting an Unit under it, will be $24\frac{1}{1}$, and $24\frac{1}{1}$ by $\frac{1}{3}$ produceth $4\frac{2}{3}$ or 16.

Quest. 9. What is the Product of 36 by $1\frac{2}{3}$?

Answer, $3\frac{2}{3}$ or $29\frac{2}{3}$.

C H A P. XXII.

Division of Vulgar Fractions.

1. IF the Dividend and the Divisor are both simple Fractions, then multiply the Numerator of the Dividend into the Denominator of the Divisor, and the Product is a new Numerator, and multiply the Denominator of the Dividend into the Numerator of the Divisor, and the Product is a new Denominator, which new Fraction thus found, is the Quotient you desire. *Example.*

Quest. 1. What is the Quotient of $\frac{1}{2}$ divided by $1\frac{1}{2}$?

Ans. $\frac{2}{3}$, or $1\frac{1}{2}$, for the first I multiply (5) the Numerator of the Dividend into (5) the Denominator of the Divisor, and the Product (25) is a Numerator for the Quotient, then I multiply (8) the

$$\begin{array}{r} 3 \overline{) 5} = 1 \text{ R } 2 \\ \underline{3} \\ 2 \\ \underline{2} \\ 0 \end{array} \quad \begin{array}{r} 5 \overline{) 8} = 1 \text{ R } 3 \\ \underline{5} \\ 3 \end{array} \quad \begin{array}{r} 25 \overline{) 24} = 1 \text{ R } 1 \\ \underline{25} \\ 1 \end{array}$$

De

Denominator of the Dividend, into (3) the Numerator of the Divisor, and the Product (24) I put in the Quotient for a Denominator, so I find $\frac{3}{4}$ is the Quotient sought.

Quest. 2. What is the Quotient of $\frac{1}{17}$ divided by $\frac{2}{3}$?

Ans. $\frac{3}{4}$ equal to $\frac{3}{4}$ in its lowest Terms.

2. But if you will divide a simple Fraction by a Compound, or a Compound by a Simple, first reduce such Compound to a simple Fraction, then go on as before.

Quest. 3. What is the Quotient of $\frac{1}{12}$ divided by $\frac{3}{4}$ of $\frac{1}{2}$?

Answer. $\frac{3}{8}$ or $\frac{3}{8}$ first reduce $\frac{3}{4}$ of $\frac{1}{2}$ into a simple Fraction, and it is $\frac{3}{8}$, by which $\frac{1}{12}$ being divided, the Quotient is $\frac{3}{8}$ equal in its least Terms to $\frac{3}{8}$, and if the Dividend and Divisor be both of Compound Fractions, reduce them both to a simple Fraction, then divide the one by the other, as in Rule 1. foregoing.

Quest. 4. What is the Quote of $\frac{3}{4}$ of $\frac{1}{2}$ divided by $\frac{1}{2}$ of $\frac{1}{2}$?

Answer. $\frac{1}{2}$ or $\frac{1}{2}$, or $\frac{1}{2}$, or $\frac{1}{2}$ in its lowest Terms.

3. If the Dividend, or Divisor, or both, are mixed Numbers, reduce them to improper Fractions, and perform Division as you are taught before.

Quest. 5. What is the Quote of $12 \frac{3}{4}$ divided by $21 \frac{1}{4}$?

Answer. $\frac{3}{4}$ for $12 \frac{3}{4}$ is equal to $\frac{51}{4}$ and $21 \frac{1}{4}$ is equal to $\frac{85}{4}$, and the Quote of $\frac{51}{4}$ divided by $\frac{85}{4}$ is as before $\frac{3}{4}$.

4. If you divide a Fraction by a whole Number, or a whole Number by a Fraction, make the whole Number an Improper Fraction, by putting an Unit for a Denominator to it, as was taught in Rule 4. Chap. 22. and then perform Division as was before taught.

Example.

Quest. 6. What is the Quote of 8 divided by $\frac{3}{4}$?

Answer. $4 \frac{2}{3}$, which is equal to $4 \frac{2}{3}$, being reduced as is before directed. See the Work in the Margent.

$$\begin{array}{r} 3 \overline{) 8} \left(\frac{40}{1 \ 3} \right. \text{ or } 13 \frac{1}{3} \end{array}$$

Quest. 7. What is the Quotient of $\frac{3}{4}$ divided by 8?

Answer. $\frac{3}{32}$ as per Margent.

$$\begin{array}{r} 8 \overline{) 3} \left(\frac{3}{5 \ 40} \right. \end{array}$$

C H A P. XXIV.

The Rule of Three Direct in Vulgar Fractions.

1. **A**S in the Rule of Three in Whole Numbers, so likewise in Fractions, you must see that the Fractions of the first and third Places be of the same Denomination.

2. If any of the given Fractions be Compound, let them be reduced to simple of the same Value.

3. If there are given mixed Numbers, reduce them to improper Fractions by the first Rule of Chap. XIX.

4. If any of the three Terms is a whole Number, make it an improper Fraction by constituting an Unit for its Denominator.

Having reduced your Fraction as is directed in the 4 last Rules, then proceed to a Resolution, which is performed the same Way as in whole Numbers, Respect being had to the Rules delivered for the working of Fractions, *viz.* Multiply the 2d and 3d Fractions together, according to the the first Rule of Chap. XXII. and divide the Product by the first Fraction, according to the first Rule of Chap. XXIII. and the Quotient is the Answer.

(Or, (which is better)

5. Multiply the Numerator of the first Fraction into the Denominator of the second and third, and the Product is a new Denominator; then multiply the Denominator of the first Fraction into the Numerator of the 2d and 3d, and the Product is a new Numerator, which new Fraction is the 4th Proportional or Answer, which (if it be an improper Fraction) must be reduced to a whole or mixed Number by the 3d Rule of Chap. XIX. *Example,*

Quest. 1. If $\frac{3}{4}$ Yards of Cloth cost $\frac{5}{8}$ l. what will $\frac{3}{4}$ Yds cost?

Having placed the given Fractions according to the 6th Rule of Chap. X. I proceed to the Resolution, and first I multiply the Numerator of the first Fraction (3) into 8 and 10, the Denominators of the second and third Fractions, and the Product is 240 for a Denominator; then

multiply 4 the *Denominator* of Yards
the first Fraction into 5 and 3
2, the Numerators of the
second and third Fractions, 4
the Product is 180 for a Nu-
merator, which Numerator *Facit*,
180, and Denominator 240
make $\frac{180}{240}$ l. for the Answer,
equal to $\frac{3}{4}$ or 15 s.

l.	Yards.	l.
3	5	9
180		
4	8	10
	1.	240
	180 equal to 3	
	240	4

Quest. 2. If $\frac{2}{3}$ l. buy $\frac{5}{8}$ Yards of Cloth, what will $\frac{1}{2}$ Yds cost at that rate?

Answer. kkk l. equal to $\frac{1}{5}$ l. or 14 s. 8 d.

Quest. 3. If $\frac{7}{8}$ l. cost $\frac{3}{4}$ s. what will $\frac{2}{5}$ s. buy?

Answer. $\frac{3}{4}$ l. equal to $1 \frac{1}{2}$ l.

Quest. 4. If $\frac{3}{4}$ of an Ell of Holland cost h of a Pound, how much will 12 h Ells cost at that rate?

Answer. $1 \frac{2}{7}$ equal to $7 \frac{1}{7}$ l.

In resolving the last Question and the two next, observe the 3d Rule of the Chapter foregoing.

Quest. 5. If $\frac{2}{5}$ of a C. cost 284 s. what will $7 \frac{1}{2}$ C. cost at that rate?

Answer. 239 $\frac{1}{2}$ s. or 11 l. 19 s. 7 d.

Quest. 6. If $3 \frac{1}{4}$ Yards of Velvet cost $3 \frac{1}{8}$ l. how much will $10 \frac{1}{2}$ Yards cost at that rate?

Answer. $11 \frac{3}{4}$ l.

Quest. 7. If five Yards of Broad-cloth cost $2 \frac{4}{5}$ l. what will $14 \frac{3}{4}$ Yards cost?

Answer. 13 l. 9 s. 4 d.

In working the last Question, and the four next, observe the 4th Rule of the Chapter foregoing.

Quest. 8. If 14 l. of Pepper cost 14 s. $6 \frac{1}{2}$ d. I demand the Price of $73 \frac{3}{4}$ l?

Answer. 3 l. 16 s. $7 \frac{4}{7}$ d.

Quest. 9. If 1 l. of Cochineel cost 1 l. 5 s. what will $36 \frac{7}{12}$ l. cost?

Answer. 47 l. 17 s. 6 d.

Quest. 10. If a Yard of Broad-cloth cost $15 \frac{1}{4}$ s. what will four Pieces, each containing $27 \frac{3}{8}$ Yards cost at that rate?

Answer. 85 l. 14 s. $3 \frac{1}{7}$ d.

Quest.

Quest. 11. A Mercer bought $3\frac{1}{2}$ Pieces of Silk, each Piece contained $24\frac{3}{4}$ Ells, at 6 s. 2 d. per Ell; I demand the Value of $3\frac{1}{2}$ Pieces at that rate?

Answer. 26 l. 3 s. $4\frac{3}{4}$ d.

In resolving the four next Questions, observe the 8th Rule of Chap. 19.

Quest. 12. If $\frac{2}{3}$ of an Ounce of Silver cost 2 s. I demand the Price of $11\frac{2}{3}$ l. at that rate?

Answer. 35 l.

Quest. 13. If $1\frac{1}{4}$ l. of Gold is worth $61\frac{1}{4}$ l. Sterling, what is a Grain worth at that rate?

Answer. $1\frac{1}{2}$ d.

Quest. 14. If $\frac{3}{4}$ Yards of Silk is worth $\frac{3}{4}$ of $\frac{5}{8}$ l. what is the Price of $15\frac{2}{3}$ Ells Flemish?

Answer. 9 l. 12 s. 6 d.

Quest. 15. If $\frac{2}{3}$ of $\frac{3}{4}$ of a Pound of Cloves cost 6 s. $2\frac{2}{3}$ d. what cost the C. weight at that rate?

Answer. 69 l. 6 s. 8 d.

Note, That when the Answer to the Question in this and the next Chapter are given in Fractions, they are given in their lowest Terms.

C H A P. XXV.

The Rule of Three Inverse in Fractions.

1. **I**T hath been already taught (in the 3d Rule of the 11th Chapter) how to discover when the 4th proportional Number (to the three given Numbers) is to be found out by a *Rule of Three Direct*, and when by a *Rule of Three Inverse*; to which Rule the Learner is now referred.

2. When (in Fractions) you find a Question to be solved by the *Rule of Three Inverse*, viz. when the third Term is the Divisor, then having reduced the Terms exactly (according to the Rules in Chap. 24.) multiply the Numerators of the three Fractions into the Denominators of the 2d and 1st Fractions, and the Product is a new Denominator; then multiply the Denominator of the 3d Fraction into the Numerators of the 2d and 1st Fractions, and the Product is a new Numerator, which new Fraction thus found, is the Answer to the Question.

Quest. 1. If $\frac{3}{4}$ of a Yard of Cloth that is two Yards wide, will make a Garment, how much of any other Drapery that is $\frac{1}{2}$ of a Yard wide will make the same Garment?

Answer. $2\frac{1}{2}$ Yards.

Quest. 2. I lent my Friend 46 l. for $\frac{4}{5}$ of a Year, how much ought he to lend me for $1\frac{7}{8}$ Parts of a Year?

Answer. 63 $\frac{1}{2}$ l.

Quest. 3. If $\frac{2}{3}$ of a Yard of Cloth that is $2\frac{1}{2}$ Yards wide will make any Garment, what Breadth is that Cloth when $1\frac{3}{4}$ Yard will make the same Garment?

Answer. $\frac{5}{8}$ of a Yard wide.

Quest. 4. How many Inches in Length of a Board that is 9 Inches broad, will make a Foot Square?

Answer. 16 Inches in Length.

Quest. 5. If when the Bushel of Wheat cost 4 s. $\frac{3}{4}$, the Penny-loaf weighed $10\frac{1}{4}$ Ounces, what will it weigh when the Bushel cost 8 s. 7?

Answer. $5\frac{1}{2}\frac{3}{4}$ Ounces.

Quest. 6. If 17 Men can mow $24\frac{1}{2}$ Acres in $10\frac{2}{3}$ Days, in how many Days will six Men do the same?

Answer. In $21\frac{1}{2}$ Days.

CHAP. XXVI.

Rules of Practice.

1. **I**N the single Rule of Three, when the first of the 3 Numbers in the Question (after they are disposed according to the sixth Rule of Chapter 10,) happeneth to be an Unit (or 1) that Question many Times may be resolved far more speedily than by the Rule of Three, which kind of Operation is commonly called *Practice*, and indeed it is of excellent Use among Merchants, Tradesmen, and others, by reason of its Speediness in finding a Resolution to such Kind of Questions.

2. The chiefest Questions resolvable by these brief Rules, may be comprehended under the three general Heads or Cases following, viz.

When

When the
given Price
of the Inte-
ger consists

- 1 Of Farthings under 4.
- 2 Of Pence under 12.
- 3 Of Pence and Farthings.
- 4 Of Shillings under 20.
- 5 Of Shillings, Pence, and Farthings.
- 6 Of Pounds.
- 7 Of Pounds, Shillings, Pence, and Farthings.

It would be very convenient for the Practical Arithmetician to have by Heart the several Products of the 9 Digits multiplied by 12, for *his* speedy reducing Pence into Shillings, and Shillings into Pence, which he may gain by the following Table.

	1	12
	2	24
	3	36
	4	48
12 Times	5	60
	6	72
	7	84
	8	96
	9	108

3. Shillings are practically reduced into Pounds thus, *viz.* Cut off the Figure standing in the Place of Units with a Dash of the Pen, and note it for Shillings, then draw a Line under the given Number, and take half the remaining Figures (after the first is cut off) and set them under the Line, and they are so many Pounds; but if the last Figure is odd, then take the lesser half and add 10 to the Figure so cut off (as before) for Shillings; as if I were to reduce 43658 Shillings into Pounds, first I cut off the last Figure (8) for Shillings, then I take half of the remaining Figures (4365) thus, half of 4 is 2, which I put under the Line, then half of 3 is 1, and because 3 is an odd Number, I make the next Figure 6 to be 16, and I go on, saying, half of 16 is 8, then half of 5 is 2, which is the last Figure, wherefore because 5 is an odd Number, I add 10 to the 8 I cut off, and it makes 18 s. so that I find it to be 2182 l. 18 s. as per Margent.

$$\begin{array}{r}
 43658 \\
 \hline
 2182 \quad l. \quad 18 \quad s.
 \end{array}$$

4. It is likewise convenient that the Learner be acquainted with the Practical Tables following the first containing the Aliquot or even Parts of a Shilling, the 2d containing the Aliquot Parts of a Pound.

	d.		
The even Parts of a Shilling.	6	} is {	$\frac{1}{2}$
	4		$\frac{1}{3}$
	3		$\frac{1}{4}$
	2		$\frac{1}{6}$
	1½		$\frac{1}{8}$
	1		$\frac{1}{12}$

	s.	d.	
The even Parts of a Pound.	10	00	} is {
	6	08	
	5	00	
	4	00	
	3	04	
	2	06	
	2	00	
	1	08	
	1	00	

Case 1.

5. When the Price of an Integer is a Farthing, then take the 6th part of the given Number, which will be so many Three-half-pences, and if any Thing remains it is Farthings by the 7th Rule of Chap. 9. then consider that Three-half-pence is $\frac{1}{4}$ of a Shilling, wherefore take the 8th Part of them for Shillings, and if any thing remain, they are so many Three-half-pences, which reduce into Pounds by the third Rule foregoing. *Example*, What comes 67486 l. to, at a Farthing per l.? First, I take $\frac{1}{6}$ of 67486, and it is 11247 Three-half pences and four Farthings, or 1 Penny, then $\frac{1}{8}$ of 11247 is 1405 s. and seven remains, which is seven Three-half-pences, or 10½ d. which, with the four Farthings before, make 11½ d. and 1405 s. which by the third Rule is 70 l. 5 s. In all 70 l. 5 s. 12 d. for the Answer. See the Work following.

$\frac{1}{6}$	67486 at $\frac{1}{4}$ per l.
	————— d.
$\frac{1}{8}$	11247 ———— 1
	—————
$\frac{1}{10}$	1405 ———— 10½
	—————
	l.—s.—d.
	70 4 11½ Facit.

Other

Other Examples follow.

$\frac{1}{8}$	8576 l.	at 1 qr.
$\frac{1}{8}$	1072	2 qrs.
$\frac{1}{20}$	176 8	8 d.
	<i>l. s. d.</i>	
	88 8 8	Facit.

$\frac{1}{3}$	6380 l.	at 1 qr.
$\frac{1}{8}$	1063	2 qrs.
$\frac{1}{20}$	13 2	11 d.
	<i>l. s. d.</i>	
	6 12 11	Facit

6. When the Price of the Integer is 2 Farthings, then take the Third Part of the given Number, for so many Three-half-pences, and the Remainder if any, is Half-pence, then take the Eight Part of that for Shillings, as before, &c.

Example.

$\frac{1}{3}$	7368 l.	at 2 qrs
$\frac{1}{8}$	2456	
$\frac{1}{20}$	30 7	
	<i>l. s.</i>	
	15 7	Facit

$\frac{1}{3}$	8347 l.	at 2 qrs.
$\frac{1}{8}$	2782	2 qrs.
$\frac{1}{20}$	34 7	9 d. $\frac{1}{2}$
	<i>l. s. d, Facit.</i>	
	17 7 9 $\frac{1}{2}$	

7. When the Price of the Integer is 3 Farthings, then take half the given Number for Three-half-pence, and if any Thing remain it is 3 Farthings; then take the 8th for Shillings, as before, &c.

$\frac{1}{2}$	4736 l.	at 5 qrs.
$\frac{1}{8}$	2368	
$\frac{1}{20}$	29 6	
	<i>l. s.</i>	
	14 16	Facit

$\frac{1}{2}$	5425 l.	at 3 qrs
$\frac{1}{8}$	2712	3 qrs.
$\frac{1}{20}$	33 9	
	<i>l. s. d. qrs.</i>	
	16 19 0 3	Facit.

Case 2.

8. When the given Price of the Integer, is a Part or Parts of a Shilling, (*viz.* Pence) divide the given Number of Integers, (whose Value is sought) by the Denominator of the Fraction representing the even Part, and the Quote is Shillings (always minding the 7th Rule

Rule of the 9th Chapter) and those Shillings may be reduced into Pounds by the third Rule of this Chapter, Example : Let it be required to find the Value of 438 *l.* at 3 *d.* per *l.* I consider 3 *d.* is $\frac{1}{4}$ of a Shilling, and 438 *l.* will cost so many 3 Pences, wherefore I divide 438 by 4 the Denominator or $\frac{1}{4}$, and the Quote is 109 Shillings, and two remains, which is two 3 *d.* or 6 *d.* the whole Value is 5 *l.* 9 *s.* 6 *d.* as by the following Work appeareth.

$$\begin{array}{r|l} \frac{1}{4} & 438 \text{ l. at } 3 \text{ d.} \\ \hline 20 & 109 \text{ --- } 6 \\ \hline \end{array}$$

1. s. d.
Facit 5 9 6

If the Learner is minded to try the Fruitfulness of his Genius, he may frame as many Examples as he thinks fit, and work them as before.

9. If the Price of the Integer be Pence under 12, and yet not an even Part, then it may be divided into even Parts, and so the Parts of the given Numbers taken accordingly, and added together, as if it were 5 *d.* which is 3 *d.* and 2 *d.* viz. $\frac{1}{4}$ and $\frac{1}{2}$ of a Shilling, first take $\frac{1}{4}$ of the given Number, and then $\frac{1}{2}$ thereof, and add them together, and their Sum is the Answer in Shillings, still observing Rule 7 of Chapter 9, for the Remainder, (if any be) then bring the Shillings into Pounds by the third Rule foregoing. Likewise 7 *d.* is $\frac{1}{2}$ and $\frac{1}{4}$, to 9 *d.* is $\frac{1}{2}$ and $\frac{1}{3}$, and 10 *d.* is $\frac{1}{2}$ and $\frac{1}{3}$, and 11 *d.* is $\frac{1}{2}$ and $\frac{2}{3}$ of a Shilling; or else many Times your Work may be shortned thus. viz. when the said given Price is to be divided into even Parts of a Shilling, or of a Pound, after you have taken the first even Part, the other may be an even Part of that Part, as in the next Example, where are given 439 *l.* at 5 *d.* per *l.* now I may divide it thus, viz. into 4 *d.* and 1 *d.* and 4 *d.* being $\frac{1}{3}$ of a Shilling, and 1 *d.* being $\frac{1}{4}$ of 4 *d.* I first take $\frac{1}{3}$ of 439 *l.* and it gives 146 *s.* 4 *d.* and for the 1 *d.* I take $\frac{1}{4}$ of 146 *s.* 4 *d.* which is 36 *s.* 7 *d.* which in all comes to 9 *l.* 2 *s.* 11 *d.* Examples follow.

	<i>l.</i>	<i>d.</i>
	439	at 5 per <i>l.</i>
$\frac{1}{3}$	146	4
$\frac{1}{4}$	36	7
	18	2 11
	9 <i>l.</i> 2 <i>s.</i> 11 <i>d.</i> Facit	
	<i>Ells</i>	<i>d.</i>
	587	at 7 per Ell
$\frac{1}{3}$	195	8
$\frac{1}{4}$	146	9
	34	2 5
	17 <i>l.</i> 2 <i>s.</i> 5 <i>d.</i> Facit	
	<i>yds.</i>	<i>d.</i>
	836	at 8 per yd.
$\frac{1}{3}$	278	8
$\frac{1}{3}$	278	8
	6	57 4
	37 <i>l.</i> 17 <i>s.</i> 4 <i>d.</i> Fac.	

	<i>yds.</i>	<i>d.</i>
	417	at 9 per yd.
$\frac{1}{2}$	201	6
	104	3
$\frac{1}{10}$	31	2 9
	15 <i>l.</i> 12 <i>s.</i> 9 <i>d.</i> Fac.	
	<i>Ells</i>	<i>d.</i>
$\frac{1}{2}$	386	at 10
$\frac{1}{3}$	193	
	128	8
	32	1 8
	16 <i>l.</i> 01 <i>s.</i> 8 <i>d.</i> Facit	
	<i>l.</i>	<i>d.</i>
	534	at 11
$\frac{1}{2}$	178	
$\frac{1}{3}$	178	
$\frac{1}{4}$	153	6
	48	9 6
	24 <i>l.</i> 9 <i>s.</i> 6 <i>d.</i> Facit.	

10. When the Price of the Integer is Pence and Farthings, if it make an even Part of a Shilling, work as before; but if they are uneven, as Penny Farthing, Penny three Farthings, 2 *d.* 1 *qr.* or 2 *d.* 3 *qrs.* 1 *d.* 3 *qrs.* or the like, then first work for some even Part, and then consider what Part the rest is of that even Part, and divide that Quotient thereby, then add them together. and

and reduce them to Pounds as before. *Example.* 3476 l. at 1 d. 1 qr. per l. first I work for the Penny by dividing 3470 l. by 12, for 1 d. is $\frac{1}{12}$ of a Shilling, and the Quote is 289 s. 2 d. then I conceive that one Farthing is the $\frac{1}{4}$ of a Penny, and the Value of one Farthing will be $\frac{1}{4}$ of the Value of a Penny, and therefore I take $\frac{1}{4}$ of 289 s. 2 d. which is 72 s. 3 d. 2 qrs. and add them together, and they are 18 l. 1 s. 5 d. 2 qrs. as by the Margent.

	l.	d.	qrs.
3470 at 1	1		
$\frac{1}{4}$	289	2	
	72	3	$\frac{1}{2}$
$\frac{1}{8}$	36	1	5
	18	1	5
			2

Case 4.

11. When the Price of the Integer is 2 s. then cut off the Figure in the Place of Units of the given Number, and double it for Shillings, and the Figures on the other Hand are Pounds. *Example,* 436 Yards at 2 s. per Yard, cut off the last Figure 6, and double it, makes 43 | 6 12 s. and the other two Figures, viz 43, are so many Pounds; so that their Value is 43 l. 12 s. 43 l. 12 s. as per Margent.

12. Hence it is evident that when the given Price of an Integer is an even Number of Shillings, then if you take half of that (even) Number of Shillings, and multiply the given Number of Integers thereby, doubling the first Figure of the Product, and setting it apart for Shillings, the rest of the Product will be Pounds, which Pounds and Shillings are the Value sought. *Example.* What cost 536 Yards at 8 s. per Yard? To resolve which, I take Half of 8 s. (the Price of a Yard) which is 4, and multiply 536 thereby, saying, 4 Times 6 is 24, then I double the first Figure 4 makes 8 536 Yds at 8 s. for Shillings, and carry 2 to the next Product, &c. I find the rest of the Product to be 214, which I note for Pounds; so that the Value of 536 Yards at 8 s. per Yard, is 214 l. 8 s. as by the Margent. Other Examples of the same Kind may be wrought after the same Manner.

13. If the given Price of the Integer is an odd Number of Shillings, then work first for the even Number

of Shillings by the last Rule, and for the odd Shilling take $\frac{1}{20}$ of the given Number of Integers, according to the 3d Rule of this Chapter, and add them together, and you have your Desire. *Examples follow.*

Tds s.
422 at 3 per Yard

<i>l.</i>	<i>s.</i>
42	4
21	2

63 6 *Facit.*

Ells s.
516 at 7 per Ell

<i>l.</i>	<i>s.</i>
154	16
25	16

180 12 *Facit*

Ells s.
431 at 13

<i>l.</i>	<i>s.</i>
298	12
21	12

280 03 *Facit*

Ells s.
324 at 17 per Ell.

<i>l.</i>	<i>s.</i>
259	04
16	04

275 08 *Facit*

14. Except when the given Price of the Integer is 5 s. for then it is sooner answered by taking $\frac{1}{4}$ of the given Number, whose Value is sought, as in the following Example.

$\frac{1}{4}$	<i>yds. s.</i>
	436 at per Yard
	109 <i>l. Facit</i>

$\frac{1}{4}$	<i>Ells s.</i>
	206 at 5 per ell
	51 <i>l. 10s. Facit</i>

Case 5.

15. When the given Price of an Integer is Shillings and Pence, or Shillings, Pence and Farthings; then divide the given Number of Integers whose Value you seek by the Denominator of that Fraction representing that even Part. As for Example, What is the Price of 384 yards at 6 s. 8 d. per yard? Here I consider that 6 s. 8 d. is $\frac{1}{3}$ of a Pound, wherefore divide 384 by 3, and the Quote is the Answer, viz. 128 l. So that 384 yards at 6 s 8 d. per yard, amounts to 128 l. as per Margent, still observing the 7th Rule of the 9th Chapter.

$\frac{1}{3}$	384
	128 <i>l. fa.</i>

16. When the given Value of the Integer is Shillings and

and Pence, and not an even Part of a Pound, yet many times it may be divided into Parts (*viz.* 6 s. 6 d. is 4 s. and 2 s. 6 d.) for the 4 s. Work according to the 12th Rule foregoing, and for the 2 s. 6 d. take the eighth Part of the given Number, and add them together, then their Sum is the Value required.

So 8 s. 6 d. will be divided into 6 s. and 2 s. 6 d. and the the Price of the given Number may be found out as before, &c. Examples follow.

<i>yds</i>	<i>s.</i>	<i>d.</i>	<i>Ells</i>	<i>s.</i>	<i>d.</i>
386 at 8	8		s. 240 at 5	4	
<hr/>			<hr/>		
$\frac{1}{2}$ 128 l. 13	4		2 54	0	
$\frac{1}{4}$ 38	12	0	$\frac{1}{2}$ 40	0	
$\frac{1}{8}$ 38	12	0	<hr/>		
<hr/>			64 l. Facit.		
167 l. 5 s. 4 d. Facit					
<i>Ells</i>	<i>s.</i>	<i>d.</i>	<i>yds</i>	<i>s.</i>	<i>d.</i>
427 at 8	6		s. 386 at 14	8	
<hr/>			<hr/>		
128 l. 2	6		8 154 l. 8	0	
53	7		$\frac{1}{2}$ 128	13	4
<hr/>			<hr/>		
181 l. 9 s. 6 d. Facit.			283 l. 1 s. 4 d. Facit.		

17. When the given Price of an Integer is Shillings and Pence, and you cannot readily divide them according to the last Rule, then multiply the given Number whose Value you seek, by the Number of Shillings in the Price of the Integer, and then for the Pence work by the 8th Rule foregoing; then add the Numbers together, and their Sum is their Value sought in Shillings; as for Example. What is the Value of 392 Yards at 6 s. 9 d. per Yard. Here 6 s. 9 d. cannot be made an even Part, nor indeed can it be divided into even Parts of a Pound; wherefore multiply the given Number of Yards 392 by 6 for the 6 s. the Product is 2352 s. then for the 9 d. I divide it into 6 d. and 3 d. and work for them by the 8th Rule foregoing and at last add the Shillings together, they make 2646 s. and by the 3d they are reduced to 132 l. 6 s. the Value of 392 yards at 6 s. 9 d. per yard. See the Work.

		—392 yds at 6 s. 9 d.
		<hr/>
	23	52
1	2	196
1	1	98
4		<hr/>
		264 6
		<hr/>
		132 l. 6 s. Facit

In like manner Variety of other Examples be wrought.

18. When the given Price of the Integer is Shillings, Pence, and Farthings, then multiply the given Number of Integers, by the Number of Shillings contained in the Value of the Integer, and for the Pence and Farthings follow the 10th Rule of this Chapter.

Example

Ells s. d.			Ells s. d.		
438 at 8 6 $\frac{3}{4}$			370 at 14 2 $\frac{1}{4}$		
<hr/>			<hr/>		
s.	3504		s.	480	
8	219		s.	370	
2		27 4 d. $\frac{1}{2}$			
$\frac{1}{4}$		<hr/>			
	375 0	4 $\frac{1}{2}$	14	3180	d.
	<hr/>		$\frac{1}{4}$	61	8
	Fac. 187 l. 10 s. 4 d. $\frac{1}{2}$		$\frac{1}{4}$	15	5
			$\frac{1}{4}$	7	8
			<hr/>		
				526 4	9 $\frac{1}{2}$
				<hr/>	
				Fac. 263 l. 4 s. 9 d. $\frac{1}{2}$	
				Ells s. d.	
				431 at 2 4 $\frac{1}{2}$	
				<hr/>	
				862	
				107	9 d.
				53	10 $\frac{1}{2}$
				<hr/>	
				102 3	7 $\frac{1}{2}$
				<hr/>	
				Fac. 51 l. 3 s. 7 d. $\frac{1}{2}$	

Case 6.

16. When the given Number of the Integers is Pounds, then multiply the Number of Integers, whose Value is sought by the Price of the Integer, and the Product is the Answer in Pounds.

Examples.

C.	l.
42	at 2 per C.

84 l. Facit

C.	l.
30	at 3 per C.

90 l. Facit

C.	l.
13	at 8 per C.

104 l. Facit

C.	l.
48	at 12 per C.

576 l. Facit

Case 7.

20. If the Price of the Integer is Pounds and Shillings, then for the Pounds work as in the last Rule, and for the Shillings as in the 12th and 13th Rules beforegoing, then add the Numbers produced from them both, and the Sum is the Value sought.

Examples.

C.	l.	s.
46	at 2	4

2l. 92 s.

4s. 9 4

101 4s. Facit

Gross	l.	s.
-------	----	----

58 at 3 7

3l. 174 s.

6s. 17 8

1s. 2 18

194 l. 6 s. Facit

Gross	l.	s.
82	at 4	10

4l. 328

10s. 41

369 l. Facit

Gross	l.	s.
-------	----	----

26 at 3 16

3 l. 78

15s. 19 10

1s. 1 6

98 l. 16 s. Facit.

21. When the given Price of an Integer consist of Pounds, Shillings, Pence, and Farthings, then work for the Shillings, Pence, and Farthings first, according to the 18th Rule of this Chapter, and find the Total Value of the given Number, as if there was no Pounds, then work

with the Pounds according to the 19th Rule of this Chapter, and add the Numbers thus found, and their Sum is the Total Value required.

Example of this Rule follow.

C.	l.	s.	d.
213 at 1	13	4	$\frac{1}{2}$
639			
213			
2569		d.	
13 s.	53	3	
3 d.	26	$7\frac{1}{2}$	
$1\frac{1}{2}$ d.			
284	8	$10\frac{1}{2}$	
1 l.	142	l. 08 s. $10\frac{1}{2}$ d.	
	113		
	355	l. 8 s. $10\frac{1}{2}$ facit	
Gross	l.	s.	d.
416 at 2	9	$3\frac{3}{4}$	
9 s.	3744		
3 d.	104		
$\frac{1}{2}$ d.	26		
	3874		
2 l.	193	l. 14 s.	
	832		
	1025	l. 14 s. facit.	

C.	l.	s.	d.
37 at 3	8	$10\frac{1}{2}$	
296 d.		8 s.	
18	6	6 d.	
9	3	3 d.	
4	$7\frac{1}{2}$	$1\frac{1}{2}$ d.	
328	$0\frac{1}{2}$ d.		
151.	8 s. $4\frac{1}{4}$ d.		
111		3 l.	
	127	l. 8 s. $4\frac{1}{2}$ d. fa.	

Gross	l.	s.	d.
48 at 3	15	$11\frac{1}{2}$	
240			
48			
700		15 s.	
24		6 d.	
16		4 d.	
6		$1\frac{1}{2}$ d.	
76	6		
38	6		
144		3 l.	
	182	l. 6 s. facit	

22. When there is given the Value of an Integer, and it is required to know the Value of many such Integers together, with $\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{3}{4}$ of an Integer, the first (by the former Rules) find out the Value of the given Number of Integers, and then for $\frac{1}{4}$ of an Integer take $\frac{1}{4}$ of the given Value of the Integer, or for $\frac{1}{2}$ take $\frac{1}{2}$ of the given Value of the Integer, and for $\frac{3}{4}$ first

first take the Half of the given Value, and then Half of that Half, setting each Part under the Precedent, then adding them together, their Sum will be the required Value of the Integers and their Parts. *Example*, What is the Value of 116 Yards, at 4 s. 6 d. per Yard? To give an Answer, First I work for the Value of 116 Yards, by the 5th Rule foregoing, and then for the half Yards, I take half of 4 s. 6 d. which is 2 s. 3 d. and add to the rest found as before, then is that Sum the total Value of 116 $\frac{1}{2}$ yds at 4 s 6 d. per Yard, which I find to amount to 26 l. 4 s 3 d. as by the Work in the Margent. And all other Examples of this Kind, are wrought the same Way.

yds.	s.	d.
116 $\frac{1}{2}$	at 4	6
<hr/>		
11 l.	12 s.	2 s.
14 l.	10 d.	2 s. 6 d.
	2 3	$\frac{1}{2}$ yards.
<hr/>		
26	4 3	Facit

Many more Questions may be stated, and several other Rules of Practice may be shewn according to the Methods of diverse Authors; but what have been delivered here, are sufficient for the Practical Arithmetician in all Cases whatsoever.

C H A P. XXVII.

The Rule of Barter.

1. **B**arter is a Rule among Merchants, which (in the Exchange of one Commodity for another) informs them so to proportion their Rates, as that neither may sustain Loss.

2. To resolve Questions in Barter, will not be difficult to him that is acquainted with the Golden Rule, or Rule of Three, it being altogether used in resolving such Questions.

Quest. 1. Two Merchants, (viz. A and B) Barter, A hath 13 C. 3 qrs. 14 l. of Pepper, at 2 l. 16 s. per C. and B hath Cotton at 9 d. per l. I demand how much B must give A for his Pepper?

Answer. 9 C. 1 qr.

First find by the Rule of Three, or the Rules of Practice foregoing, how much the Pepper is worth, saying, If

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If 1 C. cost 2 l. 16 s. what will 13 C. 3 qrs. 14 l. cost?

Answer. 38 l. 17 s.

Secondly, By the Rule of Three, say, If 9 d. buy 1 l. of Cotton, how much will 38 l. 17 s. buy?

Answer. $9\frac{1}{4}$ C. and so much Cotton must B give to A for 13 C. 3 qrs. 14 l. of Pepper, at 2 l. 16 s. per Cent. when the Cotton is worth 9 d. per l.

Quest. 2. A and B Barter, A hath 120 Yards of broad Cloth, worth 6 s. per Yard, but in the Barter he will have 8 s. per Yard; B hath Shalloon worth 4 s. per Yard. Now I demand how many Yards of Shalloon B must give A for his Broad-cloth, making his Gain in Barter equal to that of A?

Answer, 110 Yards of Shalloon.

First (as in the last Question) find out how B ought to sell his Shalloon in Barter, viz. say, If 6 s. require 8 s. what will 4 s. require?

Answer. 5 s. 4 d.

Thus you see that B must sell his Shalloon in Barter at 5 s. 4 d. if A sell his Broad-cloth at 8 s. per Yard.

It remaineth now to find out how much Shalloon B must give for 120 Yards of Broad-cloth, which resolved after the Method in the first Question of this Chapter is found to be 180, and so many Yards of Shalloon must B give A for the 120 Yards of Broad-cloth.

Quest. 3. A and B bartered, A had 14 C. of Sugar worth 6 d. per l. for which B gave him 1 C. 3 qrs. of Cinnamon, I demand how B rated his Cinnamon per l.

Answer. 4 s. per l.

Quest. 4. A and B Barter, A hath 4 Tun of Brandy, worth 37 l. 16 s. ready Money, but in Barter he hath 50 l. 8 s. per Tun, and giveth B 21 C. 2 qrs. $11\frac{1}{2}$ l. of Ginger for the 4 Tun of Brandy, I desire to know how much B sold his Ginger in Barter per C. and how much it was worth in ready Money?

Answer. For 9 l. 6 s. 8 d. in Barter, and it is worth 7 l. per Cent. in ready Money.

Quest. 5. A and B Barter, A hath 320 Dozen of Candles, at 4 s. 6 d. per Dozen, for which B giveth him 30 l. in Money, and the rest in Cotton at 8 d. per l. I demand how much Cotton he must give him more than 30 l.

Answer. 11 C. 1 qr.

CHAP.

C H A P. XXVIII.

Questions in Loss and Gain.

Q. 1. **A** Merchant bought 436 Yards Broad-cloth for 8 s. 6 d. per Yard, and selleth it again at 10 s. 4 d. per Yard; now I desire to know how much he gained in the 436 Yards?

Answer. 39 l. 12 s. 4 d.

First, Find out by the Rule of Three, or by Practice, how much the Cloth cost him at 8 s. 6 d. per Yard, which I find to be 185 l. 6 s. then by the same Rule find out how much he sold it for, viz. 225 l. 5 s. 4 d. then subtract 185 l. 6 s. which it cost him, for 225 l. 5 s. 4 d. which he sold it for, and there remaineth 39 l. 19 s. 4 d. for his Gain in the Sale thereof.

Otherwise, it may sooner be resolved thus, first find out how much he gain'd per Yard, viz. Subtract 8 s. 6 d. which he gave per Yard, from 10 s. 4 d. which he sold it for per Yard, the Remainder is 1 s. 10 d. for his Gain per Yard. Then say,

If one Yard gain 1 s. 10 d. what will 436 Yards gain? The Answer, by Practice or the Rule of Three, is 39 l. 19 s. 4 d. as was found before.

Quest. 2. A Draper bought 124 Yards of Holland-cloth which he gave 31 l. I desire to know how he must sell it per Yard to gain 10 l. 6 s. 8 d. in the whole Sale of 124 Yards.

Answer. At 6 s. 8 d. per Yard.

Add the Price which it cost him, (viz. 31 l.) to his intended Gain, (viz. 10 l. 6 s. 8 d.) the Sum is 41 l. 6 s. 8 d. Then say,

If 124 Yards require 41 l. 6 s. 8 d. what will 1 Yd require? By the Rule of Three, I find the Answer 6 s. 8 d.

Quest. 3. A Grocer bought 3 C. 1 qr. 14 l. of Cloves which cost him 2 s. 4 d. per l. and sold them for 52 l. 14 s. I desire to know how much he gained in the Whole?

Answer. 8 l. 12 s.

Quest. 4. A Draper bought 86 Kerseys for 129 l. I demand how he must sell them per Piece to gain 15 l. in selling out 100 l. at that rate? *Answer.* 1 l. 14 s. 6 d. per Piece; for,

As 100 is to 115 *l.* so is 129 *l.* to 148 *l.* 7 *s.*

So that by the Proportion above, I have found how much he must receive for the 86 Kerseys, to gain after the Rate of 15 per C. Then to find how he must sell them per Piece, I say,

As 86 Pieces are to 148 *l.* 7 *s.* so is one Piece to 1 *l.* 4 *s.* 6 *d.* which is the Number sought.

Quest. 5. A Grocer bought $4\frac{1}{2}$ C. of Pepper for 15 *l.* 17 *s.* 4 *d.* and it proving to be dammified) is willing to lose 12 *l.* 10 *s.* per Cent. I demand how he must sell it per *l.*?

Answer. 7 *d.* per *l.*

Subtract 12 *l.* 10 *s.* the Loss of 100 *l.* from 100 *l.* and there remains 87 *l.* 10 *s.* Then say,

As 100 *l.* is to 87 *l.* 10 *s.* so is 15 *l.* 17 *s.* 4 *d.* to 13 *l.* 17 *s.* 8 *d.* and so much he must sell it all for, to lose after the Rate propounded: Then to know how he must sell it per *l.* I say,

As 13 *l.* 17 *s.* 6 *d.* is to $4\frac{1}{2}$ C. so is 1 *l.* to 7 *d.*

Quest. 6. A Plummer sold 10 Fodder of Lead (the Fodder containing $19\frac{1}{2}$ C.) for 204 *l.* 10 *s.* and gained after the Rate of 12 *l.* 10 *s.* per 100 *l.* I demand how much it cost him per C.?

Answer. 18 *s.* 8 *d.*

To resolve this Question, add 12 *l.* 10 *s.* (the Gain per Cent.) to 100 *l.* and it makes 112 *l.* 10 *s.* Then say,

As 112 *l.* 10 *s.* is to 100 *l.* so is 204 *l.* 15 *s.* to 182 *l.* Which 182 *l.* is the Sum it cost him in all; then reduce your 10 Fodders to Half Hundreds, and it makes 390. Then say,

As 390 Half Hundreds is to 182 *l.* so is 2 Half Hundreds to 18 *s.* 8 *d.* the Price of two Half Hundreds, or 1 C. wt. and so much it stood him in per C. wt

Quest. 7. A Merchant bought eight Tuns of Wine, which being sophisticated, he selleth for 400 *l.* and loseth after the Rate of 12 *l.* in receiving 100 *l.* Now I demand how much it cost him per Tun, and how he selleth it per Gallon to lose after the said Rate?

Answer. It cost him 56 *l.* per Tun, and he must sell it at 3 *s.* 11 *d.* $\frac{10}{21}$ qrs. per Gallon, to lose 12 *l.* in receiving 100 *l.*

To resolve this Question, I consider in the first Place, that in receiving 100 l. he loseth 12 l. therefore 100 l. comes in for 112 l. laid out; wherefore to find out how much he laid out for the whole, I say,

As 100 l. is to 112 l. so is 400 l. to 448 l. and so much the 8 Tuns cost him: Then to find how much it cost per Tun, I say,

As 8 is to 448 l. so is 1 to 56 l. the Price it cost per Tun.

Now to find how he must sell it per Gallon, reduce the 8 Tuns into Gallons, make 2016. Then say,

As 2016 Gallons is to 400 l. so is 1 Gallon to 3 s. 11 d. $2\frac{1}{2}$ qrs. the Price he must sell it at per Gallon to lose as afore said.

Quest. 8. A Merchant bought 8 Tuns of Wine, which being sophisticated, he is willing to sell for 400 l. and loseth at that Rate 12 l. in laying out 100 l. upon the same; now I demand how much it cost him per Tun?

Here I consider that for 100 l. laid out, he received but 88 l. wherefore to find what 8 Tuns cost him, I say,

As 88 l. is to 100 l. so is 400 l. to 454 $\frac{2}{11}$ the Price it all cost him; then to find how much per Tun, I say,

As 8 is to 454 $\frac{2}{11}$, so is 1 to 56 $\frac{2}{11}$, or 56 l. 16 s. 4 d. $2\frac{1}{11}$ qrs. per Tun.

C H A P. XXIX.

Equation of Payments.

1. **E**quation of Payments is that Rule among Merchants, whereby we reduce the Times for Payment of several Sums of Money to an equated Time for Payment of the whole Debt, without Damage to Debtor or Creditor; and,

The Rule is,

2. Multiply the Sums of each particular Payment by its respective Time, then add the several Products together, and their Sum divide by the total Debt, and the Quotient thence arising is the equated Time, for the Payment of the whole Debt. *Example.*

Quest.

Quest. 1. A is indebted to B in the Sum of 130 *l.* whereof 50 *l.* is to be paid at 2 Months, and 50 *l.* at 4 Months, and the rest at 6 Months, now they agree to make one Payment of the total Sum; the Question is, What is the equated Time for Payment, without Damage to Debtor or Creditor?

To resolve this Question, I multiply each Payment by its Time, *viz.*

50 <i>l.</i> Multiply'd by 2 Mon.	produceth	100
50 <i>l.</i> Multiply'd by 4 Mon.	produceth	200
30 <i>l.</i> Multiply'd by 6 Mon.	produceth	180

The Sum of the Products is — 480

Then I divide 480 (the Sum of the Products) by 130 (the total Debt) and the Quotient is $3 \frac{1}{2}$ Months for the Time of paying the whole Debt.

Quest. 2. A Merchant hath owing him 1000 *l.* to be paid as followeth, *viz.* 600 *l.* at 4 Months, 200 *l.* at 6 Months, and the rest (which is 200 *l.*) at 12 Months, and he agreeth with the Debtor to make one Payment of the whole, I demand the Time of Payment without Damage to Debtor or Creditor?

600 <i>l.</i> Multiply'd by 4 Months	is	2400
200 <i>l.</i> Multiply'd by 6 Months	is	1200
200 <i>l.</i> Multiply'd by 12 Months	is	2400

The Sum of the Products is — 6000

and the Sum of the Products (6000) being divided by the whole Debt (1000 *l.*) quotes 6 Months for the Time of Payment of the whole Debt.

3. The Truth of the Rule is thus manifest, if the Interest of that Money which is paid by the equated Time (after it is due, be equal to the Interest of that Money (which by the equated Time) is paid so much sooner than it is due at any rate *per Cent.* then the Operation is true, otherwise not. Example.

The Proof of the Rule of Equation of Payments.

In the last Question 600 *l.* should have been paid at 4 Months, but is not discharged till 6 Months, (that is 2 Months after it is all due) wherefore its Interest of 2 Months at 6 *per Cent.* *per Annum* is 6 *l.* and then 200 *l.*

was to be paid at 6 Months, which is the equated Time for its Payment, therefore no Interest is reckoned for it; but 200 l. should have been paid at 12 Months. but is paid at 6 Months, which is 6 Months sooner than it ought, wherefore the Interest of 200 l. for 6 Months is 6 l. (accounting 6 l. per Cent. per Annum) which is equal to the Interest of 600 l. for 2 Months, wherefore the Work is right.

Quest. 3. A Merchant hath owing him a certain Sum to be discharged at 3 equal Payments, viz. $\frac{1}{3}$ at two Months $\frac{1}{3}$ at four Months, and $\frac{1}{3}$ at eight Months, the Question is, What is the equated Time for the Payment of the whole Debt?

In Questions of this Nature, (viz. where the Debt is divided into unequal Parts) each of its Parts is to be multiply'd by its Time, and the Sum of the Product is the Answer.

$\frac{1}{3}$ multiply'd by 2 Mon. produceth $\frac{2}{3}$
 $\frac{1}{3}$ multiply'd by 4 Mon. produceth $1\frac{1}{3}$
 $\frac{1}{3}$ multiply'd by 8 Mon. produceth $2\frac{2}{3}$

The Sum of the Product is $4\frac{2}{3}$
 which is $4\frac{2}{3}$ Months for the equated Time of Payment.

If instead of the Fractions representing the Parts, you had wrought by the Numbers themselves (represented by those Parts) according to the first and second Example, it would have been the same Answer; and suppose the Debt had been 90 l. then $\frac{1}{3}$ of it is 30 l. for each Payment, viz. at 2, 4, and 8 Months. Then,

30 l. multiply'd by 2 Mon. produceth 60
 30 l. multiply'd by 4 Mon. produceth 120
 30 l. multiply'd by 8 Mon. produceth 240

The Sum of the Product is 420
 which divided by 90 (the whole Debt) quoteth $4\frac{2}{3}$, or $4\frac{2}{3}$ Months as before.

Quest. 4. A Merchant oweth a Sum of Money to be paid $\frac{1}{3}$ at 5 Months, and $\frac{1}{4}$ at 8 Months, and $\frac{1}{4}$ at 10 Months, and he agreeth with his Creditor to make one total Payment; I demand the Time without Damage to Deb-

tor

for or Creditor? Work as in the last Question, and you will find the Answer to be 7 Months.

Quest. 5. *A* is indebted to *B* 640 *l.* whereof he is to pay 40 *l.* present Money, 350 *l.* at 3 Months, and the rest, (*viz.* 250 *l.* at 8 Months, and they agree to make an equated Time for the whole Payment, now I demand the Time?

In Questions of this Nature, (*viz.* where there is ready Money paid) you are multiplying to neglect the Money that is to be paid present, and work with the rest, as is before directed, and divide the Sum of the Products by the whole Debt, and the Quote is the Answer; for here 40 *l.* is to be paid present, and hath no Time allowed; and according to the Rule it should be multiplied by its Time, which is 0; therefore 40 Times 0 is 0, which neither augmenteth nor diminisheth the Dividend; wherefore to proceed (according to Direction) I say,

350 by 3 Months, produceth ————— 1050

250 by 8 Months, produceth ————— 2000

The Sum of the Product is 3050 which divided by 640, the whole Debt, the Quote is $4\frac{1}{2}$ Months, the Time of Payment.

Quest. 6. *A* is indebted to *B* in a certain Sum, half whereof is to be paid present Money, one 3d at 6 Months, and the rest at 8 Months; now I demand the equated Time for Payment of it all?

Ans. $3\frac{1}{2}$ Months is the Time of Payment.

Quest. 7. *A* is indebted to *B* 120 *l.* whereof $\frac{1}{3}$ is to be paid at 3 Months, $\frac{1}{4}$ at 6 Months, and the rest at 9 Months; What is the equated Time for the Payment of the whole Sum?

Ans. At 6 Months?

Quest. 8. *A* is indebted to *B* 420 *l.* which is due at the End of 6 Months, but *A* is willing to pay him 140 *l.* present, provided he can have the Remainder forborn so much the longer, to make Satisfaction for his Kindness; which is agreed upon; I desire to know what Time ought to be allotted for the Payment of the 280 *l.* remaining?

The Operation of this Question is left to the Learner, to try his Genius ; and who, in this Case, must have an Eye to the Rule of Three.

C H A P. XXX. E X C H A N G E.

1. **T**HE Rule of Exchange informeth Merchants how to exchange Monies, Weights, or Measures of one Country into (or for) the Monies, Weights, or Measures of another Country, and when the Rate, Reason, or Proportion betwixt the Money, Weights or Measures of different Countries is known, it will not be difficult for the Practitioner that is well acquainted with the Rule of Proportion (or Rule of Three) to resolve any Question, wherein it is required to exchange a given Quantity of the one Kind into the same Value of another Kind.

2. In Questions of Exchange there is always a Comparison made between the Coins, &c. of two Countries (or Kinds) or of more.

3. In Questions where there is a Comparison made between two Things, (whether they be Monies, Weights, &c.) of different Kinds, there may be a Solution found by a single Rule of Three, as by the following Example.

Quest. 1. A Merchant at *London* delivered 370 *l. Sterl.* to receive the same at *Paris* in *French Crowns*; the Exchange $3\frac{1}{3}$ *French Crowns per l. Sterling*, I demand how many *French Crowns* he ought to receive?

In placing the Numbers, observe the 6th Rule of the 11th Chapter, which being done, the given Number will stand thus:

$$\begin{array}{ccc} \text{l.} & \text{Crowns} & \text{l.} \\ 1 & \text{---} & 3\frac{1}{3} \text{---} & 370 \end{array}$$

and being reduced according to the Rules of the 12th Chapter, will stand thus :

$$\text{As } 1 \text{ is to } 3\frac{1}{3}, \text{ so is } 370 \text{ to } 1233\frac{1}{3}.$$

So that I conclude he ought to receive $1233\frac{1}{3}$ *French Crowns* at *Paris* for his 370 *l.* delivered at *London*.

Quest. 2. A Merchant delivered at *Amsterdam* 587 *l. Flemish*, to receive the Value thereof at *Naples* in *Ducats*, the

the exchange $4\frac{4}{7}$ Ducats per l. *Flemish*. I demand how many Ducats he ought to receive?

The Proportion is as followeth.

l. Ducats. l. Ducats.

As 1 is to $2\frac{4}{7}$, so is $2817\frac{1}{2}$ to 2817 $\frac{1}{2}$.

So I find he ought to receive 2817 $\frac{1}{2}$ Ducats at *Naples* for the 387 l. *Flemish* delivered at *Amsterdam*.

Quest. 3. A Merchant at *Florence* delivereth 2478 Ducatoons, to receive the Value at *London* in Pence, the Exchange at $53\frac{1}{2}$ *Sterl. per Ducatoon*; I demand how much *Sterling* he ought to receive?

The Proportion for Resolution is,

Duc. d. Duc. d.

As $\frac{1}{2}$ is to 102 , so is 3478 to 186073.
which is equal to 775 l. 6 $\frac{1}{2}$ for the Answer.

4. When there is a Comparison made between more than two different Coins, Weights, or Measures, there ariseth ordinarily two different Cases from such a Comparison.

1. When it is required to know how many Pieces of the first Coin, Weight or Measure, are equal in Value to a known Number of Pieces of the last Coin, Weight or Measure.

2. When it is required to find out how many Pieces of the last Coin, Weight, or Measure, are equal in Value to a given Number of the first sort of Coin, Weight, or Measure.

An Example of the first Case may be this, viz.

Quest. 4. If 150 Pence at *London* are equal to 3 Ducats at *Naples*, and $4\frac{4}{7}$ Ducats at *Naples* make $34\frac{1}{2}$ Shillings at *Brussels*; then how many Pence at *London* are equal to 139 s. at *Brussels*? *Facit*, 960 d.

The Question may be resolved by two single Rules of Three: For first, I say,

If $\frac{1}{2}$ Ducats at *Naples* make 150 d. at *London*, how many Pence will $3\frac{4}{7}$ Ducats make? *Answ.* 240 d.

By the foregoing Proportion we have discovered that $4\frac{4}{7}$ Ducats at *Naples* make 240 Pence at *London*; and by the Tenor of the Question we see that $4\frac{4}{7}$ Ducats at *Venice* make $35\frac{1}{2}$ Shillings at *Brussels*, therefore 240 d. at *London* are equal to $34\frac{1}{2}$ s. at *Brussels*, (for the Things that

are equal to one and the same Thing, are also equal to one another) wherefore we have a Way laid open to give a Solution to this Question by another Single Rule of Three, whose Proportion is,

As $34\frac{1}{2}$ s. at *Brussels* is to 240 d. at *London*, so is 131 s. at *Brussels* to 960 d. at *London*; which is the Answer to the Question.

An Example of the second Case may be this, viz.

Quest. 5. If 40 l. *Averdupois-weight* at *London* is equal to 36 l. Weight at *Amsterdam*, and 90 l. at *Amsterdam* makes 116 l. at *Dantzick*, then how many Pounds at *Dantzick*, are equal to 122 l. *Averdupois-weight* at *London*? *Answer*, $129\frac{2}{3}$ l. at *Dantzick*.

This Question is likewise answered by two single Rules of Three *viz.* First, I say,

As 36 l. at *Amsterdam* is to 46 l. at *London*.

So is 90 l. at *Amsterdam* to 100 l. at *London*.

And by the Question you find that 90 l. at *Amsterdam*, is 116 l. at *Dantzick*; and therefore 100 at *London* is likewise equal thereunto; wherefore again I say,

As 106 l. at *London* is to 116 l. at *Dantzick*.

So is 112 l. at *London* to $129\frac{2}{3}$ l. at *Dantzick*.

By-which I find that $129\frac{2}{3}$ l. at *Dantzick* are equal to 112 *Averdupois-weight* at *London*.

5. There is a more speedy Way to resolve such Questions as are contained under the two Cases before-mention'd, laid down by Mr. Kersey in the 3d Chapter of his Appendix to *Wingate's Arithmetick*, where he hath given two Rules for the Resolution of the Questions pertinent to the two said Cases.

6. But I shall lay down a general Rule for the Solution of both Cases; and 1st, Let the Learner observe the following Directions in placing of the given Terms, *viz.*

7. Let there be made two Columns, and in these Columns so place the given Terms one over the other, as that in the same Column there may not be found two Terms of the same Kind one with the other,

Having thus placed the Terms, the general Rule is,

Observe which of the said Columns, hath the most Terms placed in it, and multiply all the Terms therein continually, and place the last Product for a Dividend; then

then multiply the Terms in the other Column continually, and let the last Product be a Divisor; then divide the said Dividend by the said Divisor, and the Quotient thence arising will be the Answer to the Question.

So the Example of the first of the said Cases being again repeated, viz. If 150 Pence at *London* make 3 Ducats at *Naples*, and $4\frac{4}{7}$ Ducats at *Naples* make $34\frac{1}{2}$ Shillings at *Brussels*, then how many Pence at *London* are equal to 138 Shillings at *Brussels*?

The Term being placed according to the 7th Rule, will stand as followeth:

	A	B	
Pence at <i> Lond.</i>	150	3	Ducats at <i>Naples</i> .
Ducats at <i>Nap.</i>	$4\frac{4}{7}$	$34\frac{1}{2}$	Shillings at <i>Brussels</i> .
Shill. at <i>Bruss.</i>	138		

Having thus placed the Terms, that in neither Column there is two Terms of one Kind, then observe that the Column under A hath most Terms in it, therefore they must be multiplied together for a Dividend, viz. 150 multiplied by $4\frac{4}{7}$ produceth 3^{600} , which multiply'd by 138, produceth 4^{56800} , for a Dividend, then in the Column under B there are 3, and $34\frac{1}{2}$, which multiply'd together, produce 2^{107} for a Divisor, then having divided 4^{56800} by 2^{107} , the Quotient is 960 Pence for the Answer as before.

Again, Let the Example of the second Case be again repeated, viz. If 40 l. *Averdupois-weight* at *London* make 36 l. Weight at *Amsterdam*, and 90 l. at *Amsterdam* make 116 at *Dantzick*, then how many Pounds at *Dantzick* are equal to 112 l. *Averdupois-weight* at *London*?

The Terms being disposed according to the 7th Rule foregoing, will stand thus:

	A	B	
l. at <i>London</i>	40	36	l. at <i>Amsterdam</i> .
l. at <i>Amsterdam</i>	90	116	l. at <i>Dantzick</i> .
		112	l. at <i>London</i> .

whereby I find that the Terms under B multiply'd together, produce 497712 for a Dividend, and the Terms under A, viz. 40 and 90, produce 3600 for a Divisor, and Division being finished, the Quotient giveth $129\frac{1}{3}\frac{1}{4}\frac{1}{5}$ Pounds *Dantzick* for the Answer.

C H A P. XXXI.

Single Position.

1. **N**egative Arithmetick, called the *Rule of False*, is that by which we find out a Truth, by Numbers invented or supposed, either Single or Double.

2. The Rule of Single Position, is, when at once, viz. by one false Position, or feigned Number, we find out the true Number sought.

3. In the *Single Rule of False*, when you have made choice of your Position, work it according to the Tenor of the Question, as if it were the true Number sought; and if by the ordering your Position you find either the Result too much or too little, you may then find out the Number sought by this Proportion following, viz.

As the Result of your Position is to the Position, so is the given Number to the Number sought.

Example.

Quest. 1. A Person having about him a certain Number of Crowns, said, If a 4th, 3d, and 6th of them were added together, they would make just 45 l. now I demand the Number of Crowns he had about him?

Answer, 60 Crowns.

To resolve this Question, I suppose he had 24 Crowns (or any other Number that will admit of the like Division) now the 4th of 24 is 6, and the 3 is 8, and the 6th is 4 all which Parts, (6, 8 and 4) being added together, make but 18, but it should be 45, wherefore I say by the *Rule of Three*.

As 18 the Sum of the Parts is to the Position 24, so is 45 the given Number to 60 the true Number sought.

For the 4th of 60 is 15, and the 3d of 60 is 20, and the 6th of 60 is 10, which added together make 45.

C H A P. XXXII.

Double Position.

1. **T**H E Rule of double Position is when two false Positions are assumed to give a Resolution to the Question propounded.

2. When any Question is stated in double Position, make such a Cross as in the Margent.

3. Then make choice of any Number you think may be convenient for your working, which call your first Position, and place it at the End of the Cross at *a*; then work with this Position, as if it were the true Number sought, according to the Nature of your Question; then having found out your Error, either too much or too little, place it on that Side the *d*, then make choice of another Number of the same Denomination with the first Position (which call your second Position) and place it on the Side of the Cross at *b*; then work with this Position as with the former, and having found out your Error, either too much or too little, place it on that Side of the Cross at *c*; and then the Positions will stand at the Top of the Cross, and the Errors at the Bottom, each under his correspondent Position, and then multiply the Errors into the Position cross-wise, that is, multiply the first Position by the second Error, and the second Position by the first Error, and put each Product over its Position.

4. Having proceeded so far, then consider whether the Errors were both alike; that is, whether they are both too much, or both too little; and if they are alike; then subtract the lesser Product from the greater: and set the Remainder for a Dividend; then subtract the lesser Error from the greater, and let the Remainder be a Divisor, and the Quotient arising by this Division is the Answer to the Question.

5. But if the Errors are unlike, that is one too much, and the other too little, then add the Products of the Positions and Errors together, and their Sum shall be a Dividend, then add the Errors together, and their Sum shall be

be a Divisor, and the Quotient arising hence is the Answer.

Quest. 1. *A, B and C built a House, which cost 76 l. of which A paid a certain Sum unknown, B paid as much as A, and 10 l. over, and C as much as A and B: Now I desire to know each Man's Share in that Charge?*

Having made a Cross according to the 2d Rule, I come according to the 3d Rule to make choice of my first Position, and here I suppose *A* paid 6 l. which I put upon the Cross as you see, then *B* paid 16 l. (for its said he paid 10 l. more than *A*) and *C* paid 22 l. (for it's said he paid as much as *A* and *B*) then I add their Parts.

l.		l.
9		<i>A</i> 6
19		<i>B</i> 16
28		<i>C</i> 22
—		—
56		Sum 44
	120 168 288	
	6 X 9	
	2) (14	
	32 X 20	
	12	
76		76
56		44
—		—
20		Error 32

And they amount to 44, but it is said they paid 76 l. wherefore there is 32 too little, which I note down at the Bottom of the Cross under its Position for the first Error.

2dly, I suppose *A* paid 9 l. then *B* paid 19 l. and *C* 28 l. all which added together make 56, but they should make 76, wherefore the Error of this Position is 20, which I put at the Bottom of the Cross under its Position for the 2d Error; then I multiply the Errors and Position Crosswise, viz. 32 (the Error of the first Position) by 9 (the 2d Position) and the Product is 288. Then I multiply 20, (the Error of the second Position) by 6 (the first Position) and the Product is 120.

Then (according to the 4th Rule) I subtract the lesser Product from the greater, viz. 120 from 288, because the Errors are both alike, (viz. too little) and there remaineth

maineth 168 for a Dividend; then I subtract 22 (the lesser Error) from 32, the greater Error, and the Remainder is 12, for a Divisor; then I divide 168 by 12, and the Quotient is 14 for the Answer, which is the Share of A in the Payment.

6. Again 2dly, If the Errors had been both too big, it had had the same Effect, as appeareth by the following Work; for first, I suppose A paid 20 l. then B paid 30 l. and C 50 l. which in all is 100 l. but it should have been no more than 76, wherefore the first Error is 24 too much. Again, I suppose A paid 18 l. then B must pay 28 l. and C must pay 46 l. which in all is 92 l. but it should have been but 76.

20 A
30 B
50 C

100 Sum
76 Subtr.

320 112 432
20 X 18
8) (14
24 X 16
8

A 18
B 28
C 46

Sum 92
Subtr. 76

24 Error

Error 16.

wherefore the 2d Error is 16 too much; then I multiply 20 (the first Position) by 16 (the 2d Error) and the Product is 320; again, I multiply 18 (the 2d Position) by 24 (the first Error) and the Product is 432. Then because the Errors are both too much, I subtract 320 (the lesser Product) from 432 (the greater Product) and there remaineth 112 for a Dividend; likewise I subtract 16 (the lesser Error) from 24 (the greater Error) and the Difference is 8 for a Divisor; then perform Division, and the Quotient is 14 (as before for the Answer).

Again, 3dly, If the Errors had been the one too big, and the other too little, Respect being had to the fifth Rule foregoing, the Answer would have been the same; as thus, I take for my first Position. 6, and then the Error is 32 too little; then I take for my second Position 18, and then the Error is 16 too much; then I multiply the Positions and Errors cross-wise, and the Products are 96 and 576, and because the Errors are unlike, viz. one too big, and another too little, I add the Product 26 and 576 together.

96 672 57
6 X 16
48) (84
32 X 16
48

ther.

ther, and their Sum is 672 for a Dividend ; I likewise add the Errors 32 and 16 together, and their Sum is 48 for a Divisor ; then having finished Division, I find the Quotient to be 14, which is the Answer, as was found out at the two several Trials before.

For Proof of the Work, I say,

If A paid —————	14
Then B paid 14 and 10 (that is) —	24
Then C paid 14 and 24 (that is) —	38
	<hr style="width: 50px; margin: 0 auto;"/>

The Sum of all is — 76

which is the total Value of the Building, and equal to the given Number.

Those who desire to see the Demonstration of this Rule, let them read the 7th Chapter of Mr. *Kersey's* Appendix to Mr. *Wingate's* Arithmetick, *Petisus* in the 5th Book of his *Trigonometria*, or Mr. *Oughtred* in his *Clavis Mathematica*.

Quest. 2. Three Persons, A, B, and C, thus discoursed together concerning their Age ; quoth A, I am 18 Years of Age ; quoth B, I am as old as A and half C ; and quoth C, I am as old as you both, if your Years were added together. Now I desire to know the Age of each Person ?

Answer. A is 18, B is 54, and C is 72 Years of Age.

Quest. 3. A Father lying at the Point of Death, left to his three Sons, viz. A, B, and C, all his Estate in Money, and divideth it as followeth, viz. to A he gave half wanting 44 *l.* to B. he gave $\frac{1}{3}$ and 14 *l.* over, and to C he gave the Remainder, which was 82 *l.* less than the Share of B ; now I demand what was the Sum left, and each Man's Part ?

Answer. The Sum bequeathed was 588 *l.* wherefore A had 250 *l.* B had 210 *l.* and C had 128 *l.*

Quest. 4. Two Persons, viz. A and B had each in their Hands a certain Number of Crowns, and A said to B, If you give me one of your Crowns, I shall have five times as many as you ; and said B to him again, If you give

give me one of yours, then we shall each of us have an equal Number; now I demand how many Crowns had each Person?

Answer. A had 4, and B had two Crowns.

Quest. 5. What Number is that unto which if I add 1—4th of it self, and from the Sum subtract 1 8—th of it self, the Remainder will be 216?

Answer. 192.

Many more Questions may be added, but these well understood, will be sufficient, (even for the meanest Capacity) for the Resolution of any other Question pertinent to this Rule.

There may be an Objection made, because we have not treated particularly upon Interest and Rebate; but the Operation of such Questions being more applicable to Decimals, are omitted, till we come to acquaint the Learner therewith.

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The Honest *London Spy*, Part 1. Discovering
the base and subtle Intrigues of the Town in se-
veral witty and ingenious Dialogues between.

1. A Wheedling Town Bawd, and an Innocent
Country Lass.

2. Glisten-pipe, an Apothecary, *Spatula* a Sur-
geon, *Medicus* a Doctor, and *Ægrotus* a Patient.

3. A deluding Landlady and her Daughter,
and an honest Sailor.

4. A lascivious Mistress, and a handsome
'Prentice.

5. A Vintner's Bar-keeper, and a conceited
Fop of the Town. The whole laying open their
coining Intrigues, wicked and subtle Designs.

Director